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Hypermultiplet dependence of one-loop effective action in the $\mathcal{N} = 2$ superconformal theories

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Abstract: We study the one-loop low-energy effective action in the hypermultiplet sector for $\mathcal{N} = 2$ superconformal models. Any such a model contains an $\mathcal{N} = 2$ vector multiplet and some number of hypermultiplets. Gauge group G is assumed to be broken down to $\tilde{G} \times K$ where K is an Abelian subgroup and a background vector multiplet belongs to the Cartan subalgebra corresponding to K. We found a general expression for the low-energy effective action in the form of a proper-time integral. The leading space-time dependent contributions to the effective action are derived and their bosonic component structure is analyzed. The component action contains terms with three and four space-time derivatives of component fields and has the Chern-Simons-like form.

Keywords: Supersymmetric Effective Theories, Superspaces, Extended Supersymmetry.

Contents

1. Introduction

Four-dimensional $\mathcal{N}=2$ supersymmetric gauge theories are formulated in terms of $\mathcal{N}=2$ vector multiplet coupled to a massless hypermultiplets in certain representations R of the gauge group G. All such models possess only one-loop divergences [1−4] and can be made finite at certain restrictions on representations and field contents. In the model with n_σ hypermultiplets in representations R_σ of the gauge group G the finiteness condition has simple and universal form [1]

$$C(G)=\sum_\sigma n_\sigma T(R_\sigma), \tag{1.1}$$

where $C(G)$ is the quadratic Casimir operator for the adjoint representation and $T(R_\sigma)$ is the quadratic Casimir operator for the representation R_σ. A simplest solution to eq. (1.1) is $\mathcal{N}=4$ SYM theory where $n_\sigma=1$ and all fields are taken in the adjoint representation. It is evident that there are other solutions, e.g. for the case of SU(N) group and hypermultiplets in the fundamental representation one gets $T(R)=1/2$, $C(G)=N$ and $n_\sigma=2N$. A number of $\mathcal{N}=2$ superconformal models has been constructed in the context of AdS/CFT correspondence (see e.g [6−8] and references therein, the examples of such models and description of structure of vacuum states were discussed in details e.g. in ref. [9]). All these theories arise in $\mathcal{N}=4$ SYM theories on "orbifolds" of $AdS_5\times S^5$ with respect to discrete subgroup SU(4) of R-symmetry which breaks some number of the supersymmetries of the $\mathcal{N}=4$ SYM theory and acts nontrivially on the gauge group.

In this paper we study the structure of the low-energy one-loop effective action for the $\mathcal{N}=2$ superconformal theories. The effective action of the $\mathcal{N}=4$ SYM theory and $\mathcal{N}=2$ superconformal models in the sector of $\mathcal{N}=2$ vector multiplet has been studied by various methods [10, 11, 9, 12−15]. However a problem of hypermultiplet dependence of the effective action in the above theories was open for a long time.

The low-energy effective action containing both $\mathcal{N}=2$ vector multiplet and hypermultiplet background fields in $\mathcal{N}=4$ SYM theory was first constructed in ref. [16] and studied in more details in refs. [17, 18]. In this paper we will consider the hypermultiplet dependence of the effective action for $\mathcal{N}=2$ superconformal models. Such models are finite theories as well as the $\mathcal{N}=4$ SYM theory and one can expect that hypermultiplet dependence of the effective action in $\mathcal{N}=2$ superconformal models is analogous to one in $\mathcal{N}=4$ SYM theory. However this is not so evident. The $\mathcal{N}=4$ SYM theory is a special case of the $\mathcal{N}=2$ superconformal models, however it possesses extra $\mathcal{N}=2$ supersymmetry in comparison with generic $\mathcal{N}=2$ models. As it was noted in [16], just this extra $\mathcal{N}=2$ supersymmetry is the key point for finding an explicit hypermultiplet dependence of the effective action in $\mathcal{N}=4$ SYM theory. Therefore a derivation of the effective action for $\mathcal{N}=2$ superconformal models in the hypermultiplet sector is an independent problem.

In this paper we derive the complete $\mathcal{N}=2$ supersymmetric one-loop effective action depending both on the background vector multiplet and hypermultiplet fields in a mixed phase where both vector multiplet and hypermultiplet have non-vanishing expectation values.[1] The $\mathcal{N}=2$ supersymmetric models under consideration are formulated in harmonic superspace [21, 22]. We develop a systematic method of constructing the lower- and higher-derivative terms in the one-loop effective action given in terms of a heat kernel for certain differential operators on the harmonic superspace and calculate the heat kernel depending on $\mathcal{N}=2$ vector multiplet and hypermultiplet background superfields. We study a component form of a leading quantum corrections for on-shell and beyond on-shell background hypermultiplets and find that they contain, among the others, the terms corresponding to the Chern-Simons-type actions. The necessity of such manifest scale invariant P-odd terms in effective action of $\mathcal{N}=4$ SYM theory, involving both scalars and vectors, has been pointed out in [32]. Proposal for the higher-derivative terms in the effective action of the $\mathcal{N}=2$ models in the harmonic superspace has been given in [28]. We show how the terms in the effective action assumed in [28] can be actually computed in supersymmetric quantum field theory.

The paper is organized as follows. Section 2 is devoted to a brief formulation of the $\mathcal{N}=2$ supersymmetric models in the harmonic superspace and description of vacuum structure which we assume. Also in this section the elements of $\mathcal{N}=2$ supersymmetric background field method are introduced. In section 3 we discuss the structure of the superspace differential operator associated with the hypermultiplet dependent one-loop effective action on a given vacuum. Section 4 is devoted to a straightforward calculation of the one-loop low-energy effective action for the background fields satisfying the on-shell conditions (see Egs. (2.8)). In addition, the bosonic component effective action containing terms with four space-time derivatives of scalar components of the hypermultiplet is derived. The analogous Chern-Simons-type terms have been discussed in [28]. In section 5 we study the possible contributions to the effective action for the background hypermultiplet, which do

[1]See example of the effective action on Higgs branch for $\mathcal{N}=2$ model in refs. [19, 20]

not satisfy the on-shell condition (2.8). We show that in the purely bosonic sector the corresponding contribution contains the Chern-Simons-like terms with three space-time derivatives analogous to terms proposed in [28]. The results are summarized in section 6.

2. The model and background field splitting

The harmonic superspace approach provides manifestly covariant off-shell description of $\mathcal{N}=2$ supersymmetric field theories at classical and quantum levels (see modern state of harmonic superspace approach in [22]). $\mathcal{N}=2$ harmonic superspace has been introduced in [21] extending the standard $\mathcal{N}=2$ superspace with coordinates $z^M=(x^m,\theta^\alpha_i,\bar\theta^i_{\dot\alpha})$ $(i=1,2)$ by the harmonics $u^\pm_i$ parameterizing the two-dimensional sphere S^2: $u^{+i}u^-_i=1$, $\overline{u^{+i}}=u^-_i$.

The main advantage of harmonic superspace is that the $\mathcal{N}=2$ vector multiplet and hypermultiplet can be described by unconstrained superfields over the analytic subspace with the coordinates $\zeta^M\equiv(x^m_A,\theta^{+\alpha},\bar\theta^+_{\dot\alpha},u^\pm_i)$, where the so-called analytic basis is defined by

$$x^m_A=x^m-i\theta^+\sigma\bar\theta^--i\theta^-\sigma^m\bar\theta^+,\quad \theta^\pm_\alpha=u^\pm_i\theta^i_\alpha,\quad \bar\theta^\pm_{\dot\alpha}=u^\pm_i\bar\theta^i_{\dot\alpha}\ . \tag{2.1}$$

The $\mathcal{N}=2$ vector multiplet is described by a real analytic superfield $V^{++}=V^{++I}(\zeta)T_I$ taking values in the Lie algebra of the gauge group. A hypermultiplet [23], transforming in the representation R of the gauge group, is described by an analytic superfield $q^+(\zeta)$ and its conjugate $\tilde q^+(\zeta)$ (see the definition of conjugation in [22]). The scalar component fields $f^i(x_A)$ of the hypermultiplet and their conjugate $\bar f^i=(f_i)^\dagger$ form the SU(2) doublet. They, as well as the spinor component fields $\psi_\alpha,\bar\kappa^{\dot\alpha}$ of the hypermultiplet, appear as the lowest-order components in the $\theta^+,\bar\theta^+,u^\pm_i$ expansion of $q^+,\tilde q^+$. The $\mathcal{N}=2$ vector potential superfield V^{++} satisfies the reality condition $\widetilde{V^{++}}=V^{++}$ with respect to the generalized conjugation which is the combination of complex conjugation and the antipodal map and transforms under the gauge transformations as $\delta V^{++}=-\mathcal{D}^{++}\lambda$, where λ is an arbitrary real analytic superfield parameter. In the Wess-Zumino gauge, the superfield V^{++} has a finite number of the component fields $\phi,\bar\phi,A_m,\lambda_\alpha,\bar\lambda_{\dot\alpha},D^{(ij)}$ corresponding to field contents of $\mathcal{N}=2$ vector multiplet [24].

The classical action of $\mathcal{N}=2$ SYM theory coupled to hypermultiplets consist of two parts: the pure $\mathcal{N}=2$ SYM action and the q-hypermultiplet action in the fundamental or adjoint representation of the gauge group. Written in the harmonic superspace [21, 22] its action reads

$$S=\frac{1}{2g^2}\mathrm{tr}\int d^8z\,\mathcal{W}^2+\frac12\int d\zeta^{(-4)}q^{+f}_a(D^{++}+igV^{++})q^{+a}_f\,, \tag{2.2}$$

where we used the doublet notation $q^+_a=(q^+,-\tilde q^+)$.[2] By construction, the action (2.2) is manifestly $\mathcal{N}=2$ supersymmetric. Here $d\zeta^{(-4)}$ denotes the analytic subspace integration

[2]Hypermultiplet part of the action written in the symplectic covariant form [22] for any number n of hypermultiplets $q^+_a=(q^+,-\tilde q^+)$, $(a=1,\dots,2n)$. Then q^+_a is related to q^{+a} by the reality condition $\widetilde{q^+_a}\equiv q^{+a}=\Omega^{ab}q^+_b$, where $\Omega^{ab}=\Omega^{ba}$ is the invariant tensor of the symplectic group, $\mathrm{Sp}(N_c)$ for fundamental q^+ or $\mathrm{Sp}(N_c^2)$ for adjoint q^+ (generalization for more complicated representations looks evident) and the covariant

measure and

$$\mathcal{D}^{++} = D^{++} + iV^{++}, \quad D^{++} = \partial^{++} - 2i\theta^{+}\sigma^{m}\bar{\theta}^{+}\partial_{m}, \quad \partial^{++} \equiv u^{+i}\frac{\partial}{\partial u^{-i}}$$

is the analyticity-preserving covariant harmonic derivative. It can be shown that V^{++} is the single unconstrained analytic, $D^{+}_{(\alpha,\dot{\alpha})}V^{++} = 0$, prepotential of the pure $\mathcal{N} = 2$ SYM theory [21, 22], and all other geometrical object are determined in terms of it. So,the covariantly chiral superfield strength $\mathcal{W}$ is expressed through the (nonanalytic) real superfield V^{--} satisfying the equation

$$D^{++}V^{--} - D^{--}V^{++} + i[V^{++}, V^{--}] = 0.$$

This equation has a solution in form of the power series in V^{++} [21, 22]:

$$V^{--}(z,u) = \sum_{n=1}^{\infty} \int du_1 \dots du_n (-i)^{n+1} \frac{V^{++}(z,u_1)\dots V^{++}(z,u_n)}{(u^{+}u^{+}_{1})(u^{+}_{1}u^{+}_{2})\dots(u^{+}_{n}u^{+})},$$

and uses the harmonic distributions $\frac{1}{u^{+}_{1}u^{+}_{2}}$ or, in other words, Green's function $G^{(-1,-1)}(u_1,u_2)$ that obey the equation $\partial^{++}_{1}G^{(-1,-1)}(u_1,u_2) = \delta^{(1,-1)}(u_1,u_2)$. The rules of integration over SU(2) as well as the properties of harmonic distributions are given in refs. [21, 22]. To simplify the notation, we set $g = 1$ at the intermediate stages of the calculation. The explicit dependence on the coupling constant will be restored in the final expression for the effective action.

Now the superfield strength is defined by

$$\mathcal{W} = -\frac{1}{4}(\bar{D}^{+})^{2}V^{--}, \quad \bar{\mathcal{W}} = -\frac{1}{4}(D^{+})^{2}V^{--}. \tag{2.3}$$

One can prove that the $\mathcal{W}, \bar{\mathcal{W}}$ are gauge invariant, u-independent, $\mathcal{D}^{\pm\pm}\mathcal{W} = 0$, covariant chiral (antichiral) superfields, $\bar{\mathcal{D}}^{\pm}_{\dot{\alpha}}\mathcal{W} = 0$, and satisfy the Bianchi identities $(\mathcal{D}^{\pm})^{2}\mathcal{W} = (\bar{\mathcal{D}}^{\pm})^{2}\bar{\mathcal{W}}$. For further use we will write down also the superalgebra of gauge covariant derivatives with the notation $\mathcal{D}^{\pm}_{(\alpha,\dot{\alpha})} = \mathcal{D}^{i}_{(\alpha,\dot{\alpha})}u^{\pm}_{i}$:

$$\begin{gathered}
\{\mathcal{D}^{+}_{\alpha}, \mathcal{D}^{-}_{\beta}\} = -2i\varepsilon_{\alpha\beta}\bar{\mathcal{W}}, \quad \{\bar{\mathcal{D}}^{+}_{\dot{\alpha}}, \bar{\mathcal{D}}^{-}_{\dot{\beta}}\} = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}\mathcal{W}, \\
\{\bar{\mathcal{D}}^{+}_{\dot{\alpha}}, \mathcal{D}^{-}_{\alpha}\} = -\{\mathcal{D}^{+}_{\alpha}, \bar{\mathcal{D}}^{-}_{\dot{\alpha}}\} = 2i\mathcal{D}_{\alpha\dot{\alpha}}, \\
[\mathcal{D}^{\pm}_{\alpha}, \mathcal{D}_{\beta\dot{\beta}}] = \varepsilon_{\alpha\beta}\bar{\mathcal{D}}^{\pm}_{\dot{\beta}}\bar{\mathcal{W}}, \quad [\bar{\mathcal{D}}^{\pm}_{\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = \varepsilon_{\dot{\alpha}\dot{\beta}}\mathcal{D}^{\pm}_{\beta}\mathcal{W}, \\
[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = \tfrac{1}{2i}\{\varepsilon_{\alpha\beta}\bar{\mathcal{D}}^{+}_{\dot{\alpha}}\bar{\mathcal{D}}^{-}_{\dot{\beta}}\bar{\mathcal{W}} + \varepsilon_{\dot{\alpha}\dot{\beta}}\mathcal{D}^{-}_{\alpha}\mathcal{D}^{+}_{\beta}\mathcal{W}\} = \tfrac{1}{2i}\{\varepsilon_{\alpha\beta}\bar{F}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}}F_{\alpha\beta}\}.
\end{gathered} \tag{2.4}$$

The operators $\mathcal{D}^{+}_{\alpha}$ and $\bar{\mathcal{D}}^{+}_{\dot{\alpha}}$ strictly anticommute

$$\{\mathcal{D}^{+}_{\alpha}, \mathcal{D}^{+}_{\beta}\} = \{\bar{\mathcal{D}}^{+}_{\dot{\alpha}}, \bar{\mathcal{D}}^{+}_{\dot{\beta}}\} = \{\mathcal{D}^{+}_{\alpha}, \bar{\mathcal{D}}^{+}_{\dot{\alpha}}\} = 0. \tag{2.5}$$

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derivative is denoted as $\mathcal{D}^{++}q^{+a} = D^{++}q^{+a} + i\mathbf{V}^{++a}_{\ \ \ b}q^{+b}$, $\mathbf{V}^{++a}_{\ \ \ b} = V^{++I}(\mathbf{T}_I)^{a}{}_{b}$, $\mathbf{T}_I = \begin{pmatrix} -T_I^T & 0 \\ 0 & T_I \end{pmatrix}$. We also introduce the flavors as N_f-dimensional vector field q^{+f} so that q^{+f}_{a} is $N_c \times N_f$ matrix.

It follows from (2.5) that $\mathcal{D}^+_{(\alpha,\dot\alpha)} = e^{-ib} D^+_{(\alpha,\dot\alpha)} e^{ib}$ for some Lie-algebra-valued real superfield $b(z,u)$ known as the bridge [21, 22]. In the so-called λ-frame defined by $\Phi^{(p)} \to e^{ib}\Phi^{(p)}$ the gauge covariant derivatives $\mathcal{D}^+_{(\alpha,\dot\alpha)}$ coincide with the flat derivatives $D^+_{(\alpha,\dot\alpha)}$ and therefore any covariant analytic superfield becomes a function over the analytic subspace. A full set of gauge covariant derivatives includes also the harmonic derivatives $(\mathcal{D}^{++}, \mathcal{D}^{--}, \mathcal{D}^0)$, which form the algebra $su(2)$ and satisfy the obviously commutation relations with $\mathcal{D}^\pm_\alpha$ and $\bar{\mathcal{D}}^\pm_{\dot\alpha}$. Unlike $\mathcal{D}^+_{(\alpha,\dot\alpha)}$ the gauge covariant derivatives $\mathcal{D}^{\pm\pm}$ acquire connections $V^{\pm\pm}$.

The action (2.2) possesses the superconformal symmetry SU$(2,2|2)$ which is manifest in the harmonic superspace approach [22]. The low energy effective action at a generic vacuum of $\mathcal{N}=2$ gauge theory includes only massless U(1) vector multiplets and massless neutral hypermultiplets, since charged vectors and charged hypermultiplets get masses by the Higgs mechanism. The moduli space of vacua for the theory under consideration is specified by the following conditions [29]

$$[\bar\phi, \phi] = 0, \quad \phi f_i = 0, \quad \bar f^i \bar\phi = 0 \quad \bar f^{(i} T_I f^{j)} = 0\ . \tag{2.6}$$

Here the $\phi, \bar\phi$ are the scalar components of $\mathcal{N}=2$ vector multiplet and complex scalars f_i are the scalar components of the hypermultiplet.

The structure of a vacuum state is characterized by solutions to eqs. (2.6). These solutions can be classified according to the phases or branches of the gauge theory under consideration [29]. In the pure Coulomb phase $f_i = 0$, $\phi \neq 0$ and unbroken gauge group is $\mathrm{U}(1)^{\mathrm{rank(G)}}$. In the pure Higgs phase $f_i \neq 0$ and the gauge symmetry is completely broken; there are no massless gauge bosons. It is well known that F- and D-flatness conditions describing the Higgs branch can be mapped [30] to the ADHM constraints determining the moduli space of instantons. In the mixed phases, i.e. on the direct product of the Coulomb and Higgs branches (some number of $\phi, \bar\phi$ is not equal to zero and some number of f_i is not equal to zero) the gauge group is broken down to $\tilde G \times K$ where K is some Abelian subgroup and rank$(\tilde G)$ is reduced in comparison with rank(G).

Further we follow [9] and impose the special restrictions on the background $\mathcal{N}=2$ vector multiplet and hypermultiplet. They are chosen to be aligned along a fixed direction in the moduli space vacua; in particular, their scalar fields should solve eqs. (2.6):

$$V^{++} = \mathbf{V}^{++}(\zeta) H, \quad q^+ = \mathbf{q}^+(\zeta)\Upsilon\ . \tag{2.7}$$

Here H is a fixed generator in the Cartan subalgebra corresponding to Abelian subgroup K, and Υ is a fixed vector in the R-representation space of the gauge group, where the hypermultiplet takes values, chosen so that $H\Upsilon = 0$ and $\bar\Upsilon \mathbf{T}_I \Upsilon = 0$. At this point we use notations from [9]. Eq. (2.7) defines a single U(1) vector multiplet and a single hypermultiplet which is neutral with respect to the U(1) gauge subgroup generated by H. The freedom in the choice of H and Υ can be reduced by requiring that the field configuration (2.7) to be invariant under the maximal unbroken gauge subgroup.

At the tree level and energies below the symmetry breaking scale, we have free field massless dynamics of the $\mathcal{N}=2$ vector multiplet and the hypermultiplet aligned in a particular direction in the moduli space of vacua. Thus the low energy propagating fields

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are massless neutral hypermultiplets and U(1) vector which form the on shell superfields possessing the properties

$$(D^{\pm})^2 \mathcal{W} = (\bar{D}^{\pm})^2 \bar{\mathcal{W}} = 0\,, \tag{2.8}$$

$$D^{++} q^{+a} = (D^{--})^2 q^{+a} = D^{--} q^{-a} = 0, \quad q^{-a} = D^{--} q^{+a}, \quad D^{-}_{(\alpha,\dot{\alpha})} q^{-a} = 0\,.$$

All the notations are given in [22]. The equations (2.8) eliminate the auxiliary fields and put the physical fields on shell.

At the quantum level, however, exchanges of virtual massive particles produce the corrections to the action of the massless fields. The manifestly $\mathcal{N} = 2$ supersymmetric Feynman rules in harmonic superspace have been developed in [21] (see also [22, 25]). We quantize the $\mathcal{N} = 2$ supergauge theory in the framework of the $\mathcal{N} = 2$ supersymmetric background field method [12, 4] by splitting the fields V^{++}, q^{+a} into the sum of the background fields V^{++}, q^{+a}, parameterized according to (2.7), and the quantum fields v^{++}, Q^{+a} and expanding the Lagrangian in a power series in quantum fields. Such a procedure allows us to find the effective action for arbitrary $\mathcal{N} = 2$ supersymmetric gauge model in a form preserving the manifest $\mathcal{N} = 2$ supersymmetry and classical gauge invariance in quantum theory. The original infinitesimal gauge transformations are realized by two different ways: first as the background transformations:

$$\delta V^{++} = -\mathcal{D}^{++}\lambda, \quad \delta v^{++} = i[\lambda, v^{++}]\,, \tag{2.9}$$

and second as the quantum transformations

$$\delta V^{++} = 0, \quad \delta v^{++} = -\mathcal{D}^{++}\lambda - i[v^{++}, \lambda]. \tag{2.10}$$

In the background-quantum splitting, the classical action of the pure $\mathcal{N} = 2$ SYM theory can be shown to be given by

$$\begin{aligned} S_{\mathrm{SYM}}[V^{++} + v^{++}] = \; & S_{\mathrm{SYM}}[V^{++}] + \frac{1}{4}\int d\zeta^{(-4)} du v^{++} (D^{+})^2 \mathcal{W}_\lambda \\ & - \mathrm{tr}\int d^{12}z \sum_{n=2}^{\infty} \frac{(-ig)^{n-2}}{n} \int du_1 \ldots du_n \frac{v_\tau^{++}(z, u_1) \ldots v_\tau^{++}(z, u_n)}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}\,. \end{aligned} \tag{2.11}$$

$\mathcal{W}_\lambda$ and v_τ^{++} denote the λ- and τ-frame forms of $\mathcal{W}$ and v^{++} respectively $\mathcal{W}_\lambda = e^{ib}\mathcal{W}e^{-ib}$, $v_\tau^{++} = e^{-ib} v^{++} e^{ib}$. The quantum part of the action depends on V^{++} via the dependence of v_τ^{++} on b, latter being a complicated function of V^{++}. The hypermultiplet action becomes

$$\begin{aligned} S_H(q + Q) = \; & S_H[q] + \int d\zeta^{(-4)} du Q_a^+ \mathcal{D}^{++} q^{+a} + \frac{1}{2}\int d\zeta^{(-4)} du q_a^+ i v^{++} q^{+a} \\ & + \frac{1}{2}\int d\zeta^{(-4)} du \{ Q_a^+ \mathcal{D}^{++} Q^{+a} + Q_a^+ i v^{++} q^{+a} + q_a^+ i v^{++} Q^{+a} + Q_a^+ i v^{++} Q^{+a} \}\,. \end{aligned} \tag{2.12}$$

The terms linear in v^{++} and q^{+} in (2.11), (2.12) determines the equation of motion and this term should be dropped when considering the effective action.

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To construct the effective action, we will follow the Faddeev-Popov Ansatz. Within the framework of the background field method, we should fix only the quantum transformation (2.10). In accordance with [12], let us introduce the gauge fixing function in the form

$$\mathcal{F}^{(4)}_\tau = D^{++}v^{++}_\tau = e^{-ib}(\mathcal{D}^{++}v^{++})e^{ib} = e^{-ib}\mathcal{F}^{(4)}e^{ib}\,,$$

which changes by the law

$$\delta\mathcal{F}^{(4)}_\tau = e^{-ib}\{\mathcal{D}^{++}(\mathcal{D}^{++}\lambda + i[v^{++},\lambda])\}e^{ib} \tag{2.13}$$

under the quantum transformations (2.10). Eg. (2.13) leads to the Faddeev-Popov determinant

$$\Delta_{FP}[v^{++},V^{++}] = \mathrm{Det}(\mathcal{D}^{++}(\mathcal{D}^{++}+iv^{++})).$$

To get a path-integral representation for $\Delta_{\mathrm{FP}}[v^{++},V^{++}]$, we introduce two real analytic fermionic ghosts $\mathbf{b}$ and $\mathbf{c}$, in the adjoint representation of the gauge group, and the corresponding ghost action

$$S_{\mathrm{FP}}[\mathbf{b},\mathbf{c},v^{++},V^{++}] = \mathrm{tr}\int d\zeta^{(-4)}du\,\mathbf{b}\mathcal{D}^{++}(\mathcal{D}^{++}\mathbf{c}+i[v^{++},\mathbf{c}]). \tag{2.14}$$

As a result, we arrive at the effective action $\Gamma[V^{++},q^+]$ in the form

$$\begin{aligned} e^{i\Gamma[V^{++},\,q^+]} = {}& e^{iS_{cl}[V^{++},\,q^+]}\int \mathcal{D}v^{++}\mathcal{D}Q^+\mathcal{D}\mathbf{b}\mathcal{D}\mathbf{c}\,\times \\ & \times e^{i(\Delta S_{\mathrm{SYM}}[v^{++},V^{++}]+\Delta S_H[v^{++},V^{++},Q^+,q^+]+S_{\mathrm{FP}}[\mathbf{b},\mathbf{c},v^{++},V^{++}])}\delta[\mathcal{F}^{(4)}-f^{(4)}]\,, \end{aligned} \tag{2.15}$$

where $f^{(4)}(\zeta,u)$ is an external Lie-algebra valued analytic superfield independent of V^{++}, and $\delta[\mathcal{F}^{(4)}]$ is the proper functional analytic delta-function. To transform the path integral for $\Gamma[V^{++},\ q^+]$ into a more useful form, we average the right hand side in eq. (2.15) with the weight

$$\Delta[V^{++}]\exp\{\frac{i}{2\alpha}\mathrm{tr}\int d^{12}z du_1 du_2 f^{(4)}_\tau(z,u_1)\frac{(u^-_1u^-_2)}{(u^+_1u^+_2)^3}f^{(4)}_\tau(z,u_2)\}. \tag{2.16}$$

Here α is an arbitrary gauge parameter. The functional $\Delta[V^{++}]$ should be chosen from the condition

$$1 = \Delta[V^{++}]\int\mathcal{D}f^{(4)}\exp\{\frac{i}{2\alpha}\mathrm{tr}\int d^{12}z du_1 du_2 f^{(4)}_\tau(z,u_1)\frac{(u^-_1u^-_2)}{(u^+_1u^+_2)^3}f^{(4)}_\tau(z,u_2)\}. \tag{2.17}$$

Hence, using the standard identity $\int d\zeta^{(-4)}(D^+)^4L(z,u) = \int d^{12}zL(z,u)$,

$$\begin{aligned} \Delta^{-1}[V^{++}] &= \int\mathcal{D}f^{(4)}\exp\{\frac{i}{2\alpha}\mathrm{tr}\int d\zeta^{(-4)}_1 d\zeta^{(-4)}_2 du_1du_2 f^{(4)}(\zeta_1,u_1)A(1,2)f^{(4)}(\zeta_2,u_2)\} \\ &= \mathrm{Det}^{-1/2}A\,, \end{aligned} \tag{2.18}$$

we have the expression for Δ by means of a special background-dependent operator $A = \frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3}(D_1^+)^4(D_2^+)^4\delta^{12}(z_1 - z_2)$ acting on the space of analytic superfields with values in the Lie algebra of the gauge group. Thus

$$\Delta[V^{++}] = \mathrm{Det}^{1/2} A \ . \tag{2.19}$$

To find Det$A$ we represent it by a functional integral over analytic superfields of the form

$$\mathrm{Det}^{-1} A = \int \mathcal{D}\chi^{(4)}\mathcal{D}\rho^{(4)} \exp\{i\mathrm{tr}\int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 \chi^{(4)}(1) A(1,2)\rho^{(4)}(2)\}\,, \tag{2.20}$$

and perform the following change of functional variables

$$\rho^{(4)} = (\mathcal{D}^{++})^2\sigma, \quad \mathrm{Det}(\frac{\delta\rho^{(4)}}{\delta\sigma}) = \mathrm{Det}(\mathcal{D}^{++})^2 \ .$$

Then we have[3]

$$\begin{aligned} \mathrm{tr}\int d\zeta_1^{(-4)} du_1 d\zeta_2^{(-4)} du_2 \chi^{(4)}(1) A(1,2)\rho^{(4)}(2) \\ &= \mathrm{tr}\int d^{12}z du_1 du_2 \chi_\tau^{(4)}(1)\frac{(u_1^- u_2^-)}{(u_1^+ u_2^+)^3}(D_2^{++})^2\sigma_\tau(2) \\ &= \frac{1}{2}\mathrm{tr}\int d^{12}z du \chi_\tau^{(4)}(D^{--})^2\sigma_\tau \\ &= -\mathrm{tr}\int d\zeta^{(-4)} du \chi^{(4)}\widehat{\Box}\sigma\,, \end{aligned} \tag{2.21}$$

where

$$\widehat{\Box} = -\frac{1}{2}(\mathcal{D}^+)^4(\mathcal{D}^{--})^2 \ . \tag{2.22}$$

On the basis of Egs. (2.19), (2.21) one obtains

$$\Delta[V^{++}] = \mathrm{Det}^{-1/2}(\mathcal{D}^{++})^2 \mathrm{Det}^{1/2}\widehat{\Box}_{(4,0)} \ . \tag{2.23}$$

Now, we are able to represent $\Delta[V^{++}]$ by the following functional integral

$$\Delta[V^{++}] = \mathrm{Det}^{1/2}\widehat{\Box}_{(4,0)} \int \mathcal{D}\varphi e^{-\frac{i}{2}\mathrm{tr}\int d\zeta^{(-4)} du \mathcal{D}^{++}\varphi\mathcal{D}^{++}\varphi}\,, \tag{2.24}$$

with the integration variable φ being a bosonic real analytic superfield taking values in the Lie algebra of the gauge group. The φ is, in fact, the Nielsen-Kallosh ghost for the theory. As a result, we see that the $\mathcal{N} = 2$ SYM theory is described within the background field approach by three ghosts: the two fermionic ghosts **b** and **c** and the third bosonic ghost φ. The ghost action S_{FP} and S_{NK} given by eqs. (2.14) and (2.24) correspond to the known ω-multiplet [21, 22].

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[3] we use $(D_2^{++})^2\frac{u_1^- u_2^-}{(u_1^+ u_2^+)^3} = (u_2^+ u_1^-)(D_2^{--})^2\delta^{(3,-3)}(u_2, u_1)$

Upon averaging the effective action with the weight (2.16), one gets the following path integral representation

$$e^{i\Gamma[V^{++},\,q^+]} = e^{iS_{cl}[V^{++},\,q^+]}\mathrm{Det}^{1/2}\widehat{\Box}_{(4,0)} \times \int \mathcal{D}v^{++}\mathcal{D}Q^+\mathcal{D}\mathbf{b}\mathcal{D}\mathbf{c}\mathcal{D}\varphi e^{iS_q[v^{++},Q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},q^+]}, \tag{2.25}$$

where

$$S_q[v^{++},Q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},q^+] = \Delta S_{\rm SYM}[v^{++,V^{++}}] + S_{\rm GF}[v^{++},V^{++}] + \Delta S_H[v^{++},V^{++},Q^+,q^+] + S_{\rm FP}[\mathbf{b},\mathbf{c},v^{++},V^{++}] + S_{\rm NK}[\varphi,V^{++}]. \tag{2.26}$$

Here $S_{\rm GF}[v^{++},V^{++}]$ is the gauge fixing contribution to the quantum action

$$\begin{aligned} S_{\rm GF}[v^{++},V^{++}] &= \frac{1}{2\alpha}\mathrm{tr}\int d^{12}zdu_1du_2\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}(D_1^{++}v_\tau^{++}(1)(D_2^{++}v^{++}(2))) \\ &= \frac{1}{2\alpha}\mathrm{tr}\int d^{12}zdu_1du_2\frac{v_\tau^{++}(1)v_\tau^{++}(2)}{(u_1^+u_2^+)^2} - \frac{1}{4\alpha}\mathrm{tr}\int d^{12}zduv_\tau^{++}(D^{--})^2v_\tau^{++}\ . \end{aligned} \tag{2.27}$$

Let us consider the sum of the quadratic part in v^{++} of $\Delta S_{\rm SYM}$ (2.11) and $S_{\rm GF}$ (2.27). It has the form

$$\frac{1}{2}(1+\frac{1}{\alpha})\mathrm{tr}\int d^{12}zdu_1du_2\frac{v_\tau^{++}(1)v_\tau^{++}(2)}{(u_1^+u_2^+)^2} + \frac{1}{2\alpha}\mathrm{tr}\int d^{12}zduv^{++}\widehat{\Box}v^{++}\,,$$

where we have used eq. (2.22). To further simplify the computation, we make the simplest choice the Fermi-Feynman gauge $\alpha = -1$. We can now write the final result for the effective action $\Gamma[V^{++},q^+]$

$$e^{i\Gamma[V^{++},\,q^+]} = e^{iS_{cl}[V^{++},\,q^+]}\mathrm{Det}^{1/2}\widehat{\Box}_{(4,0)}\int \mathcal{D}v^{++}\mathcal{D}Q^+\mathcal{D}\mathbf{b}\mathcal{D}\mathbf{c}\mathcal{D}\varphi e^{iS_q[v^{++},Q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},q^+]}, \tag{2.28}$$

where action S_q is as follows

$$S_q[v^{++},Q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},q^+]=S_2[v^{++},Q^+,\mathbf{b},\mathbf{c},\varphi,V^{++},q^+]+S_{\rm int}[v^{++},Q^+,\mathbf{b},\mathbf{c},V^{++},q^+], \tag{2.29}$$

$$\begin{aligned} S_2 &= -\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}duv^{++}\widehat{\Box}v^{++} + \mathrm{tr}\int d\zeta^{(-4)}du\mathbf{b}(\mathcal{D}^{++})^2\mathbf{c} + \frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}du\varphi(\mathcal{D}^{++})^2\varphi \\ &\quad +\frac{1}{2}\int d\zeta^{(-4)}du\{Q_a^+\mathcal{D}^{++}Q^{+a} + Q_a^+iv^{++}q^{+a} + q_a^+iv^{++}Q^{+a}\}\,, \\ S_{\rm int} &= -\mathrm{tr}\int d^{12}zdu_1\ldots du_n\sum_{n=3}^{\infty}\frac{(-i)^{n-2}}{n}\frac{v_\tau^{++}(z,u_1)\ldots v_\tau^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)} \\ &\quad -i\mathrm{tr}\int d\zeta^{(-4)}du\mathcal{D}^{++}\mathbf{b}[v^{++},\mathbf{c}] + \frac{1}{2}\int d\zeta^{(-4)}duQ_a^+iv^{++}Q^{+a}\ . \end{aligned} \tag{2.30}$$

This equations completely determine the structure of the perturbation expansion for calculating the effective action $\Gamma[V^{++},q^+]$ of the $\mathcal{N}=2$ SYM theory with hypermultiplets in a manifestly supersymmetric and gauge invariant form. However not everybody hidden rigid symmetry of a classical action can always be maintained manifestly in the Faddeev-Popov quantization scheme. According to the analysis of [26], the problem of keeping the rigid symmetries manifest at the quantum level is essentially equivalent to finding covariant gauge conditions. In the case of conformal symmetry, such gauge conditions do not exist and any special conformal transformation has to be accompanied by a field-dependent nonlocal gauge transformation in order to restore the gauge slice [27, 9]. The invariance of the path integral under combined conformal and gauge transformations lead to modified conformal Ward identities for the effective action.

The action S_2 defines the propagators depending on background fields [12]. In the framework of the background field formalism in $\mathcal{N}=2$ harmonic superspace there appear three types of covariant matter and gauge field propagators. Associated with $\widehat{\Box}$ is a Green's function $G^{(2,2)}(z,z')$ which is subject to the Feynman boundary conditions and satisfies the equation $\widehat{\Box}\, G^{(2,2)}(1|2) = -\mathbf{1}\delta^{(2,2)}(1|2)$, where the analytic delta-function[4] $\delta^{(2,2)}(\zeta_1,\zeta_2)$ is

$$i < v^{++}(z,u)v^{++}(z',u') >= G^{(2,2)}(z,u,z',u') = -\frac{1}{\widehat{\Box}}(\mathcal{D}^+)^4\{\mathbf{1}\delta^{12}(z-z')\delta^{(-2,2)}(u,u')\} \ .$$

Sometimes, it is useful to rewrite $G^{(2,2)}$ in a manifestly analytic at both points, following [22]

$$G^{(2,2)}(1,2) = -\frac{1}{2\,\widehat{\Box}_1\widehat{\Box}_2}(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4\{\mathbf{1}\delta^{12}(z_1-z_2)(D_2^{--})^2\delta^{(-2,2)}(u_1,u_2)\} \ . \qquad (2.31)$$

This representation may, in principle, be advantageous when handling those supergraphs which contain a product of harmonic distributions.

The Q^+ hypermultiplet propagator associated with the action (2.30) has the form

$$i < Q^+(\zeta_1,u_1,\zeta_2,u_2) >= G_b^{a(1.1)}(1|2) = -\delta_b^a\frac{(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4}{(u_1^+u_2^+)^3}\frac{1}{\widehat{\Box}_1}\delta^{12}(z_1-z_2) \ . \qquad (2.32)$$

It is not hard to see that this manifestly analytic expression is the solution of the equation $\mathcal{D}_1^{++}G^{(1,1)} = \delta_A^{(3,1)}(1|2)$. For the hypermultiplet of the second type described by a chargeless real analytic superfield $\omega(\zeta,u)$ the equation for Green' function is $(\mathcal{D}_1^{++})^2G^{(0,0)}(1|2) = \delta_A^{(4,0)}(1|2)$. The suitable expression for $G^{(0,0)}$ is

$$i < \omega(1),\omega^T(2) >= G^{(0,0)}(1|2) = -\frac{1}{\widehat{\Box}_1}(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4\{\mathbf{1}\delta^{12}(z_1-z_2)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}\}. \qquad (2.33)$$

Switching off the gauge background superfield, the Green's functions (2.31, 2.32, 2.33) turn into the free ones obtained in [21, 22]. The operator $\widehat{\Box} = -\frac{1}{2}(\mathcal{D}^+)^4(\mathcal{D}^{--})^2$ transforms each covariantly analytic superfield into a covariantly analytic and, using algebra (2.4), can be rewritten as second-order d'Alemberian-like differential operator on the space of such superfields [12]:

$$\begin{aligned}\widehat{\Box} = {} & \frac{1}{2}\mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha} + \frac{i}{2}(\mathcal{D}^{+\alpha}\mathcal{W})\mathcal{D}_\alpha^- + \frac{i}{2}(\bar{\mathcal{D}}_{\dot\alpha}^+\bar{\mathcal{W}})\bar{\mathcal{D}}^{-\dot\alpha} + \frac{1}{2}\{\mathcal{W},\bar{\mathcal{W}}\} \\ & -\frac{i}{4}(\bar{\mathcal{D}}^+\bar{\mathcal{D}}^+\bar{\mathcal{W}})\mathcal{D}^{--} + \frac{i}{8}[\mathcal{D}^+,\mathcal{D}^-]\mathcal{W}\,,\end{aligned} \qquad (2.34)$$

[4] $\delta^{(q,4-q)}(\zeta_1,u_1|\zeta_2,u_2) = (D_1^+)^4\delta^{12}(z_1-z_2)\delta^{(q-4,4-q)}(u_1,u_2) = (D_2^+)^4\delta^{12}(z_1-z_2)\delta^{(q,-q)}(u_1,u_2)$ [21].

Among the important properties of $\widehat{\Box}$ is the following: $(\mathcal{D}^+)^4 \,\widehat{\Box} = \widehat{\Box}\, (\mathcal{D}^+)^4$. The coefficients of this operator depend on background superfields $\mathcal{W}, \bar{\mathcal{W}}$. For the background, belonging to an Abelian subgroup of the gauge group and satisfying the on-shell conditions, we have the further restriction: $\mathcal{D}^{\pm}\mathcal{W} = D^{\pm}\mathcal{W}$, and similarly for $\bar{\mathcal{W}}$ with $D, \bar{D}$ being background independent derivatives. In addition, we should omit the two last terms in (2.34) since they disappear on-shell.

To simplify the quadratic part of the action for quantum gauge superfields it is convenient to expand these superfields in some basis. We choose the quantum superfields in one-to-one correspondence with the roots of the Lie algebra of gauge group G: $v = \sum_\alpha v^\alpha E_\alpha + \sum_i v^i H_i$. Here E_α is the generator corresponding to the root α normalized as $\mathrm{tr}(E_\alpha E_{-\beta}) = \delta_{\alpha,-\beta}$ and H_i are the rank(G) Cartan subalgebra generators satisfying the commutation relations $[H_i, E_\alpha] = \alpha(H_i)E_\alpha$. Using these notations one can rewrite the actions (2.30) in terms of coefficients in expansions of v in above basis. Such form of writing of the effective action is very convenient for its evaluation and will be used in section 4 for various cases.

3. Structure of the one-loop effective action

Consider the loop expansion of the effective action within the background field formulation. Then, the effective action is given by vacuum diagrams (that is diagrams without external lines) with background field dependent propagators and vertices. A formal expression of the one-loop effective action $\Gamma[V^{++},\, q^+]$ for the theory under consideration is written in terms of a path integral as follows [12]:

$$e^{i\Gamma[V^{++},\, q^+]} = e^{iS(V^{++},\, q^+)}\mathrm{Det}^{1/2}\,\widehat{\Box}_{(4.0)} \int \mathcal{D}v^{++}\mathcal{D}\mathbf{b}\mathcal{D}\mathbf{c}\mathcal{D}\varphi\mathcal{D}Q^+ \; e^{iS_2[v^{++},\mathbf{b},\mathbf{c},\varphi,Q^+,V^{++},q^+]}\,, \tag{3.1}$$

where the full quadratic action is defined in eq. (2.30):

$$S^{(2)}[v^{++},\mathbf{b},\mathbf{c},\varphi,Q^+,V^{++},q^+] = -\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}du\, v^{++}\,\widehat{\Box}\, v^{++} + \mathrm{tr}\int d\zeta^{(-4)}du\,\mathbf{b}(\mathcal{D}^{++})^2\mathbf{c} \tag{3.2}$$
$$+\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}du\,\varphi(\mathcal{D}^{++})^2\varphi - \frac{1}{2}\int d\zeta^{(-4)}(Q^{a+}\mathcal{D}^{++}Q^+_a + q^{+a}iv^{++}Q^+_a + Q^{+a}iv^{++}q^+_a)\,.$$

Here v^{++} is a quantum vector superfield taking values in the Lie algebra of the gauge group and $\mathbf{b}$, $\mathbf{c}$ are two real analytic Faddeev-Popov fermionic ghosts and φ is the bosonic Nielsen-Kallosh ghost, all in the adjoint representation of the gauge group. Eqs. (3.1), (3.2) completely determine the structure of the perturbation expansion for calculating the effective action of the $\mathcal{N}=2$ SYM with hypermultiplets in a manifestly supersymmetric and gauge invariant form. For the propagators of the quantum vector multiplet v^{++} and the hypermultiplets Q^{+a} we use (2.31) and (2.32) respectively. Vertices can be taken directly from the second line (3.2). It is easy to see that the ghosts do not couple to background hypermultiplet and therefore do not contribute to hypermultiplet dependent part of the one-loop effective action. In the vector sector of the $\mathcal{N}=2$ SYM theory where the matter hypermultiplet are integrated out, the one-loop effective action $\Gamma[V^{++}]$ reads

$$\Gamma[V^{++}] = \frac{i}{2}\mathrm{Tr}_{(2,2)} \ln \widehat{\Box} - \frac{i}{2}\mathrm{Tr}_{(4,0)} \ln \widehat{\Box} - \frac{i}{2}\mathrm{Tr}_{ad} \ln(\mathcal{D}^{++})^2 + i\mathrm{Tr}_{R_q} \ln \mathcal{D}^{++} + \frac{i}{2}\mathrm{Tr}_{R_\omega} \ln(\mathcal{D}^{++})^2.$$

Currently, the holomorphic and non-holomorphic parts of the low-energy effective action $\mathcal{N}=2,4$ SYM theory on the Coulomb branch, including Heisenberg-Euler type action in the presence of a covariantly constant vector multiplet, are completely known (see for a review e.g. [10, 11]). The general structure of the low-energy effective action in $\mathcal{N}=2,4$ superconformal theories is [13]:

$$\begin{aligned} \Gamma = S_{cl} + \int d^{12}z\{c \ln \mathcal{W} \ln \bar{\mathcal{W}} + \int d^{12}z \ln \mathcal{W} \Lambda(\frac{D^4 \ln \mathcal{W}}{\bar{\mathcal{W}}^2}) + c.c. \\ + \int d^{12}z \Upsilon(\frac{\bar{D}^4 \ln \bar{\mathcal{W}}}{\mathcal{W}^2}, \frac{D^4 \ln \mathcal{W}}{\bar{\mathcal{W}}^2})\} + \dots , \end{aligned} \tag{3.3}$$

where Λ and Υ are holomorphic and real analytic function of the (anti)chiral superconformal invariants. The c-term is known to generate four-derivative quantum corrections at the component level which include an famous F^4 term (see e.g. [11]).The hypermultiplet dependent part of the effective action in $\mathcal{N}=4$ SYM theory in leading order is also known [16] - [18].

For further analysis of the effective action it is convenient to diagonalize the action of quantum fields $S^{(2)}$ using a special shift of hypermultiplet variables in the path integral[5]

$$\begin{aligned} Q^{+a} &= \xi^{+a} + i\int d\zeta_2^{(-4)} q^{+b}(2) v^{++}(2) G_b^{a(1.1)}(1|2)\,, \\ Q_a^{+} &= \xi_a^{+} - i\int d\zeta_2^{(-4)} G_a^{b(1.1)}(1|2) v^{++}(2) q_b^{+}(2)\,, \end{aligned} \tag{3.4}$$

where ξ^{+a}, ξ_a^{+} are the new independent variables in the path integral. It is evident that the Jacobian of the replacement (3.4) is equal to unity. Here $G_b^{a(1.1)}(1|2)$ is the background-dependent propagator (2.32) for the superfields Q^{+a}, Q_b^{+}. In terms of the new set of quantum fields we obtain for the following hypermultiplet dependent part of the quadratic action

$$S_H^{(2)} = -\frac{1}{2}\int d\zeta^{(-4)} \xi^{a+} \mathcal{D}^{++} \xi_a^{+} - \frac{1}{2}\int d\zeta_1^{(-4)} d\zeta_2^{(-4)} q^{+a}(1) v^{++}(1) G_a^{b(1.1)}(1|2) v^{++}(2) q_b^{+}(2)\ .$$

Then the vector multiplet dependent part of the quadratic action gets the following non-local extension

$$S_v^{(2)} = -\frac{1}{2}\mathrm{tr}\int d\zeta_1^{(-4)} v_1^{++} \int d\zeta_2^{(-4)} \left(\widehat{\Box}\delta_A^{(2.2)}(1|2) + q^{+a}(1) G_a^{b(1.1)}(1|2) q_b^{+}(2)\right) v_2^{++}\ . \tag{3.5}$$

Expression (3.5), written as an analytical nonlocal superfunctional, will be a starting point for our calculations of the one-loop effective action in the hypermultiplet sector. Our aim in the current and later sections is to find the leading low-energy contribution to the

[5] In QED such a change of variables has been fulfilled in ref. [31]. N.G.P is grateful to S. Kuzenko for bringing his attention to this reference.

effective action for the slowly varying hypermultiplet when all derivatives of the background hypermultiplet can be neglected. We will show that for such a case the non-local interaction is localized.

Using the relation $v_2^{++} = \int d\zeta_3^{(-4)}\delta_A^{(2.2)}(2|3)v_3^{++}$ one can rewrite expression for $S_v^{(2)}$ (3.5) in the form

$$S_v^{(2)} = -\frac{1}{2}\mathrm{tr}\int d\zeta_1^{(-4)}\, v_1^{++}\int d\zeta_2^{(-4)}\Big(\widehat{\Box}\delta_A^{(2.2)}(1|2) \\ + \int d\zeta_3^{(-4)} q^{+a}(1)G_a^{b(1.1)}(1|3)q_b^{+}(3)\delta_A^{(2.2)}(3|2)\Big)\, v_2^{++}\ . \qquad (3.6)$$

Then we use the explicit form of the Green function (2.32) and the relation allowing us to express the $(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4$ as a polynomial in powers of $(u_1^+u_2^+)$ [14]

$$(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4 = (\mathcal{D}_1^+)^4\left((\mathcal{D}_1^-)^4(u_1^+u_2^+)^4 - \frac{i}{2}\Delta_1^{--}(u_1^+u_2^+)^3(u_1^-u_2^+) - \widehat{\Box}_1(u_1^+u_2^+)^2(u_1^-u_2^+)^2\right)\ , \qquad (3.7)$$

where the operator Δ^{--} is [14]

$$\Delta^{--} = \mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_\alpha^-\bar{\mathcal{D}}_{\dot\alpha}^- + \frac{1}{2}\mathcal{W}(\mathcal{D}^-)^2 + \frac{1}{2}\bar{\mathcal{W}}(\bar{\mathcal{D}}^-)^2 + (\mathcal{D}^-\mathcal{W})\mathcal{D}^- + (\bar{\mathcal{D}}^-\bar{\mathcal{W}})\bar{\mathcal{D}}^-\ . \qquad (3.8)$$

Since $G^{(1.1)}(1,2) = -G^{(1.1)}(2,1)$, the non-local term in (3.6) takes the form

$$\int d\zeta_3^{(-4)} q^{+a}(1)(\mathcal{D}_3^+)^4\left((\mathcal{D}_3^-)^4(u_3^+u_1^+)\frac{1}{\widehat{\Box}_3} - \right. \qquad (3.9) \\ \left. -\frac{i}{2}\Delta_3^{--}(u_3^-u_1^+)\frac{1}{\widehat{\Box}_3} - \frac{(u_3^-u_1^+)^2}{(u_3^+u_1^+)}\right)\delta^{12}(1|3)q_a^+(3)\delta_A^{(2.2)}(3|2)\ .$$

The large braces here contain three terms. It is easy to see that two first terms include the derivatives which will lead to derivatives of the hypermultiplet in the effective action. Since we keep only contributions without derivatives, the above terms can be neglected. As a result, is it sufficient to consider only the third term in the braces.

Now we apply the relation $\int d\zeta_3^{(-4)}(\mathcal{D}_3^+)^4 = \int d^{12}z$, allowing to integrate over z_3, and obtain

$$-\int du_3\, q^{+a}(1)\frac{(u_3^-u_1^+)^2}{(u_3^+u_1^+)}q_a^+(u_3,z_1)\delta_A^{(2.2)}(u_3,z_1|2)\ .$$

Then one uses the on-shell harmonic dependence of hypermultiplet $q^{+a}(3) = u_{3i}^+q^{ia}$ and take the coincident limit $u_1 = u_3$ (conditioned by $\delta_A^{(2.2)}(u_3,z_1|2)$). After that we get $\int du_3\frac{u_{3i}^+}{u_3^+u_1^+} = -u_{1i}^-$. As a result, the term under consideration has the form

$$q^{+a}(1)q_a^-(1)\delta_A^{(2.2)}(1|2),$$

where the expression $q^{+a}(1)q_a^-(1) = q^{ia}q_{ia}$ is treated further as the slowly varying superfield and all its derivatives are neglected. Namely such an expression was obtained in [18] by summation of harmonic supergraphs.

Thus, the second term in (3.6) becomes local in the leading low-energy approximation. As a result, the operator in action $S_v^{(2)}$ determining the effective background covariant propagator of the quantum vector multiplet superfield v_I^{++} (we expanded the superfield v^{++} in generators $v^{++} = v_I^{++}T_I$ and work further only with superfield components v_I^{++}) takes the form

$$\left(\widehat{\Box}_{IJ} + q^{+a}(z_1,u_1)\{T_I,T_J\}q_a^-(z_1,u_1)\right)\delta_A^{(2.2)}(1|2)\,, \tag{3.10}$$

where

$$\widehat{\Box}_{IJ} = \mathrm{tr}\left(T_{(I}\Box T_{J)} + \frac{i}{2}T_{(I}[\mathcal{D}^{+\alpha}\mathcal{W},T_{J)}]\mathcal{D}^-_\alpha + \frac{i}{2}T_{(I}[\bar{\mathcal{D}}^+_{\dot\alpha}\bar{\mathcal{W}},T_{J)}]\bar{\mathcal{D}}^{-\dot\alpha} + T_{(I}[\mathcal{W},[\bar{\mathcal{W}},T_{J)}]]\right)\,.$$

Here $\Box = \frac{1}{2}\mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}$ is the covariant d'Alemberian.

Thus, using the $\mathcal{N}=2$ harmonic superspace formulation of the $\mathcal{N}=2$ SYM theory with hypermultiplets and techniques of the non-local shift we obtained that the whole dependence on the background hypermultiplet is concentrated in the quantum vector multiplet sector with the modified quadratic action. Therefore the one-loop effective action is given by the expression

$$\Gamma^{(1)}[V^{++},q^+] = \Gamma_v^{(1)}[V^{++},q^+] + \widetilde{\Gamma}^{(1)}[V^{++}]\,, \tag{3.11}$$

where the first term in (3.11) is originated from quantum vector multiplet v_I^{++}

$$\Gamma_v^{(1)}[V^{++},q^+] = \frac{i}{2}\mathrm{Tr}\ln(\widehat{\Box}_{IJ} + q^{+a}\{T_I,T_J\}q_a^-)\,. \tag{3.12}$$

Second term in (3.11) is the contribution of ghosts and quantum hypermultiplet ξ_a^+ and does not depend on the background hypermultiplet.

As a result, the background hypermultiplet dependence of one-loop effective action is included into the operator

$$\widehat{\Box}_{IJ} + q^{+a}\{T_I,T_J\}q_a^-\,, \tag{3.13}$$

acting on v_I^{++} and containing the mass matrix of the vector multiplet

$$(\mathcal{M}_v^2)_{IJ} = \mathrm{tr}\left([T_I,\mathcal{W}][\bar{\mathcal{W}},T_J] + (I\leftrightarrow J)\right) + q^{+a}\{T_I,T_J\}q_a^-\,, \tag{3.14}$$

if q^+ is in the fundamental representation, and

$$(\mathcal{M}_v^2)_{IJ} = \mathrm{tr}\left([T_I,\mathcal{W}][\bar{\mathcal{W}},T_J] + [q^{+a},T_I][T_J,q_a^-]\right) + (I\leftrightarrow J)\,, \tag{3.15}$$

if q^+ in an arbitrary matrix representation. We have proved that the hypermultiplet dependence is completely transferred into the sector of quantum superfields v^{++} and conditioned by the background covariant operator (3.13). Eqs. (3.11), (3.12) are a starting point for calculating the one-loop effective action. Note that we make no restrictions on a space-time dependence of the hypermultiplet except the on-shell properties (2.8).

In the above discussion, the gauge group structure of the superfields $\mathcal{W}, q_a^+$ has been completely arbitrary. Henceforth, the background superfields will be chosen to be aligned along a fixed direction in the moduli space of vacua in such a way that their scalar fields should solve Egs. (2.6). Let the background vector multiplet and hypermultiplet be of the form (2.7) where H is a fixed generator in the Cartan subalgebra. It corresponds to assumption that gauge group G is broken down to $\tilde{G} \times K$ where K is an Abelian subgroup conditioned by the Cartan subalgebra where the generator H belongs to. In this case there is a single vacuum combination $\mathcal{W}\bar{\mathcal{W}}$ for $\mathcal{N} = 2$ background vector multiplet and a single vacuum combination $q^{+a}q^-_a$ for the background hypermultiplet.[6] Then the operator acting on the quantum vector multiplet superfields defined in (3.13) takes the universal form

$$\Box + \frac{i}{2}\alpha(H)(\mathcal{D}^+\mathcal{W}\mathcal{D}^- + \bar{\mathcal{D}}^+\bar{\mathcal{W}}\bar{\mathcal{D}}^-) + \alpha^2(H)\mathcal{W}\bar{\mathcal{W}} + q^{+a}q^-_a Z\,, \tag{3.16}$$

where $\Box$ is the covariant d'Alemberian, the combination $q^{+a}q^-_a$ $(a = 1, 2)$ already has no matrix indices (since a fixed direction in moduli space is taken) and the matrix Z has the indices I, J conditioned by the expression $\{T_I, T_J\}$ after fixation of the background hypermultiplet in accordance with (2.7). All matrices containing $\mathcal{W}, \bar{\mathcal{W}}$ in (3.16) are diagonal over the indices of the generators in $\tilde{G}$.

We are interesting only in the hypermultiplet dependent terms in the one-loop effective action (3.12). Let us clarify how such terms can in principle be generated in (3.12). The mass matrix has the structure $\mathcal{M}^{(2)}_v = \alpha^2(H)\mathcal{W}\bar{\mathcal{W}} \cdot Y + q^{+a}q^-_a \cdot Z$. The only eigenvalue of the matrix Y is 1 with $n(H)$ corresponding eigenvectors. The matrix in parentheses in (3.16) has the same eigenvectors as Y. As to the matrix Z, there can be two options:

i) The matrix Z has $n(\Upsilon)$ eigenvectors common with eigenvectors of Y $(n(\Upsilon) \leq n(H))$ and the corresponding eigenvalues are $r(\Upsilon)$. Then the effective action is the sum over different values of $r(\Upsilon)$. Therefore we assume, without loss of generality, that there is only one eigenvalue $r(\Upsilon)$ with $n(\Upsilon)$ eigenvectors common with eigenvectors of Y. Hence the hypermultiplet dependent effective action in the case under consideration is

$$\begin{aligned}\Gamma^{(1)}_v[V^{++}, q^+] = \\ \frac{i}{2}n(\Upsilon)\mathrm{Tr}\ln\left(\Box + \frac{i}{2}\alpha(H)(\mathcal{D}^+\mathcal{W}\mathcal{D}^- + \bar{\mathcal{D}}^+\bar{\mathcal{W}}\bar{\mathcal{D}}^-) + \alpha^2(H)\mathcal{W}\bar{\mathcal{W}} + r(\Upsilon)q^{+a}q^-_a\right).\end{aligned} \tag{3.17}$$

Here Tr means the functional trace of operators acting on analytic superfields of the appropriate U(1) charge.[7] The eigenvectors of Y which do not coincide with eigenvectors of Z and give no hypermultiplet dependent contributions to the effective action.

[6] If the background fields corresponds to various generators H_i the effective action will be a sum of contributions over the index i where each contribution has the structure corresponding to the above case. Therefore we can consider the case with fixed generator H without loss of generality.

[7] Specifically, if $\mathcal{A}^{(p,\,4-p)}(\zeta_1, \zeta_2)$ is the kernel of an operator acting on space of covariantly analytic superfields of charge p, then

$$\mathrm{Tr}\mathcal{A}^{(p,\,4-p)} = \mathrm{tr}\int d\zeta^{(-4)}du\mathcal{A}^{(p,\,4-p)}(\zeta, \zeta),$$

where the trace $'\mathrm{tr}'$is over group indices.

ii) The matrices Y and Z have no common eigenvectors. The effective action is the sum over eigenvectors of Y and eigenvectors of Z. The contribution originated from the eigenvectors of Y are hypermultiplet independent. The contributions originated from the eigenvectors of Z do not contain the operators $\mathcal{D}^-$ and $\bar{\mathcal{D}}^-$ since the corresponding matrix in (3.13) has different eigenvectors then Y. However these operators are used to obtain, in principle, the non-zero low-energy effective action. Therefore in this case the hypermultiplet dependent part of the effective action vanishes.

As the result, the hypermultiplet dependent effective action is given by the expression (3.17). In the next section we will consider the evaluation of this expression.

4. Calculation of the one-loop effective action

The expression (3.17) is a basis for an analysis of the hypermultiplet dependence of the effective action. This expression will be written in the convenient form allowing us to evaluate it using the superfield proper time techniques (see [34, 35] for $\mathcal{N}=1$ superfield proper time techniques) by generalizing the Schwinger construction [36]. We will follow the generic approach developed in our paper [18] where hypermultiplet dependence of $\mathcal{N}=4$ SYM effective action was analyzed.

In the framework of the Fock-Schwinger proper-time representation, the effective action (3.17) is written as follows

$$\begin{aligned}\Gamma^{(1)}_v[V^{++},q^+] &= \frac{i}{2}n(\Upsilon)\int d\zeta^{(-4)}du\int_0^\infty \frac{ds}{s}e^{-s(\Box+\frac{i}{2}\alpha(H)(\mathcal{D}^+\mathcal{W}\mathcal{D}^-+\bar{\mathcal{D}}^+\bar{\mathcal{W}}\bar{\mathcal{D}}^-)+\mathcal{M}_v^2)}\times\\ &\qquad\times(\mathcal{D}^+)^4\left(\delta^{12}(z-z')\delta^{(-2,2)}(u,u')\right)|_{z=z',u=u'}\\ &=\int_0^\infty\frac{ds}{s}\mathrm{Tr}K(s),\end{aligned} \tag{4.1}$$

where $\mathcal{M}_v^2=\alpha^2(H)\mathcal{W}\bar{\mathcal{W}}+r(\Upsilon)q^{+a}q^-_a$. Here $K(s)$ is a superfield heat kernel, the operation Tr means the functional trace in the analytic subspace of the harmonic superspace $\mathrm{Tr}K(s)=\mathrm{tr}\int d\zeta^{(-4)}K(\zeta,\zeta|s)$, where tr denotes the trace over the discrete indices. Representation of the effective action (4.1) allows us to develop a straightforward evaluation of the effective action in a form of covariant spinor derivatives expansion in the superfield Abelian strengths $\mathcal{W},\bar{\mathcal{W}}$. The leading low-energy terms in this expansion correspond to the constant space-time background $D^-_\alpha D^+_\beta\mathcal{W}=\mathrm{const}$, $\bar{D}^-_{\dot\alpha}\bar{D}^+_{\dot\beta}\bar{\mathcal{W}}=\mathrm{const}$ and on-shell background hypermultiplet. Especially we want to emphasize that on-shell conditions do not mean that the hypermultiplet is constant. Furthermore we assume that hypermultiplet is a slowly varying function in the superspace and neglect any derivatives of the hypermultiplet for deriving the superfield effective action. However, it does not mean that we miss all space-time derivatives in the component effective Lagrangian. Grassmann measure in the integral over harmonic superspace $d^4\theta^+d^4\theta^-$ generates four space-time derivatives in component expansion of the superfield Lagrangian. Therefore the above assumption is sufficient to obtain a component effective Lagrangian including four space-time derivatives of the scalar components of the hypermultiplet. Possible contributions to the hypermultiplet dependent effective action off-shell will be discussed in the next section.

Calculation of the effective action (4.1) is based on evaluating the superfield heat kernel $K(s)$. Note that even with regard to the properties of a non-analytic integrand, the crucial idea [15] is to stay in the analytic subspace at all stages of the calculations without an artificial conversion of the analytic integral into the full superspace integral, where, as a rule, an integrand contains ill-defined products of harmonic distributions. In this respect, integration with the analytic measure will be a high-power projector which removes all harmonic singularities. Moreover, because the covariant d'Alemberian doesn't contain efficiently acting $\mathcal{D}^+$ we, during heat kernel treatment, never obtain $\mathcal{D}^+q^-$ but only $\mathcal{D}^-q^- = 0$. Under this approach we should actually look for higher-derivative quantum corrections in the form

$$\int d\zeta^{(-4)}(\mathcal{D}^+)^4\mathcal{H}(\mathcal{W},\bar{\mathcal{W}},q^+,q^-). \tag{4.2}$$

In the case of a covariantly constant hypermultiplet $\mathcal{D}_m q^+ = 0$ and vector multiplet $\mathcal{D}_m\mathcal{W} = \mathcal{D}_m\bar{\mathcal{W}} = 0$ the heat kernel can be computed exactly. In order to obtain the complete kernel it is convenient separate of the contributions of the diamagnetic and paramagnetic parts of the operator $\overset{\frown}{\Box}$. We follow here a generic scheme of calculations [18] taking into account only the aspects essential for the theory under consideration. As the first step we use the Baker-Campbell-Hausdorff relation and write the operator $K(s)$ as a products of several operator exponents[8]

$$\begin{aligned} K(s) &= \exp(-s\{A^+\mathcal{D}^- + \bar{A}^+\bar{\mathcal{D}}^- + \frac{1}{2}\mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha} + \mathcal{M}_v^2\}) \\ &= \exp\{-f_{\alpha\dot\alpha}(s)\mathcal{D}^{\alpha\dot\alpha}\}\exp\{-s\frac{1}{2}\mathcal{D}^{\alpha\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\}\exp\{-\Omega(s)\}\exp\{-s(A^+\mathcal{D}^- + \bar{A}^+\bar{\mathcal{D}}^-)\}\ . \end{aligned} \tag{4.3}$$

with some unknown coefficients in the right hand side. These coefficients can by found directly by solving the system of a differential equations on these coefficients. To find the mentioned system of equations we consider $(\frac{d}{ds}K(s))K^{-1}(s)$ and substitute for $K(s)$ first and second lines in (4.3) subsequently. Equations for the function $f^{\dot\alpha\alpha}(s)$ have the form

$$\begin{aligned} \frac{d}{ds}f_{\alpha\dot\alpha}(s) &= -f_{\beta\dot\beta}F^{\dot\beta\beta}_{\dot\alpha\alpha} - A^{+\beta}(D^-_\beta f_{\alpha\dot\alpha}) - \bar{A}^{+\dot\beta}(\bar{D}^-_{\dot\beta} f_{\alpha\dot\alpha}) \\ &\quad + A^+_\beta\bar{A}^-_{\dot\beta}(\int_0^s d\tau e^{\tau F})^{\dot\beta\beta}_{\dot\alpha\alpha} + \bar{A}^+_{\dot\beta}A^-_\beta(\int_0^s d\tau e^{\tau F})^{\dot\beta\beta}_{\dot\alpha\alpha}\ . \end{aligned} \tag{4.4}$$

It is easy to show that the solution of these equation can be written as follows

$$f_{\alpha\dot\alpha} = -A^+_\delta\mathcal{F}^{\delta\dot\delta}_{\alpha\dot\alpha}\bar{A}^-_{\dot\delta} - \bar{A}^+_{\dot\delta}\bar{\mathcal{F}}^{\delta\dot\delta}_{\alpha\dot\alpha}A^-_\delta, \tag{4.5}$$

[8] Where we have used the notations

$$A^{+\alpha} = \frac{i}{2}\alpha(H)(\mathcal{D}^{+\alpha}\mathcal{W}), \quad \bar{A}^{+\dot\alpha} = -\frac{i}{2}\alpha(H)(\bar{\mathcal{D}}^{+\dot\alpha}\bar{\mathcal{W}}), \quad \mathcal{N}^\beta_\alpha = D^-_\alpha A^{+\beta}, \quad \bar{\mathcal{N}}^{\dot\beta}_{\dot\alpha} = \bar{D}^-_{\dot\alpha}\bar{A}^{+\dot\beta}$$

where the function $\mathcal{F}(\mathcal{N},\bar{\mathcal{N}},s),\bar{\mathcal{F}}(\mathcal{N},\bar{\mathcal{N}},s)$ are listed in [18]. Analogously, equation for the function Ω is

$$\frac{d}{ds}\Omega(s)-\mathcal{M}_v^2 = -A^{+\alpha}(D^-_\alpha\Omega)-\bar{A}^{+\dot{\alpha}}(\bar{D}^-_{\dot{\alpha}}\Omega)+A^+_\alpha f^{\alpha\dot{\alpha}}\bar{A}^-_{\dot{\alpha}}+\bar{A}^+_{\dot{\alpha}}f^{\dot{\alpha}\alpha}A^-_\alpha \\ -\frac{1}{2}A^+_\beta\bar{A}^-_{\dot{\beta}}(\int_0^s d\tau e^{-\tau F})^{\dot{\beta}\beta}_{\dot{\alpha}\alpha}F^{\dot{\alpha}\alpha}_{\dot{\rho}\rho}f^{\dot{\rho}\rho}-\frac{1}{2}\bar{A}^+_{\dot{\beta}}A^-_\beta(\int_0^s d\tau e^{-\tau F})^{\dot{\beta}\beta}_{\dot{\alpha}\alpha}F^{\dot{\alpha}\alpha}_{\dot{\rho}\rho}f^{\dot{\rho}\rho}. \tag{4.6}$$

Solution of these equation has the form

$$\Omega(s) = s\mathcal{M}_v^2+A^{+\alpha}\Omega^-_\alpha(s)+\bar{A}^{+\dot{\alpha}}\bar{\Omega}^-_{\dot{\alpha}}(s)+(A^+)^2\Psi^{(-2)}(s)+(\bar{A}^+)^2\bar{\Psi}^{(-2)}(s) \\ +A^{+\alpha}\bar{A}^+_{\dot{\alpha}}\Psi^{\dot{\alpha}(-2)}_\alpha(s). \tag{4.7}$$

We point out that this solution is a finite order polynomials in power of the Grassmannian elements $A^\pm,\bar{A}^\pm$. All coefficients are given in ref. [18]. Now it is necessary to write the last exponential in (4.3) in the form

$$\exp\{-s(A^+\mathcal{D}^-+\bar{A}^+\bar{\mathcal{D}}^-)\} = 1+a^{+\alpha}\mathcal{D}^-_\alpha+\bar{a}^{+\dot{\alpha}}\bar{\mathcal{D}}^-_{\dot{\alpha}}+f^{+2}(\mathcal{D}^-)^2+\bar{f}^{+2}(\bar{\mathcal{D}}^-)^2 \\ +f^{+2\dot{\alpha}\alpha}\mathcal{D}_\alpha\bar{\mathcal{D}}_{\dot{\alpha}}+\bar{\Xi}^{+3\dot{\alpha}}\bar{\mathcal{D}}^-_{\dot{\alpha}}(\mathcal{D}^-)^2+\Xi^{+3\alpha}\mathcal{D}^-_\alpha(\bar{\mathcal{D}}^-)^2 \\ +\Omega^{+4}(\mathcal{D}^-)^2(\bar{\mathcal{D}}^-)^2. \tag{4.8}$$

The coefficients of this expansion can be found exactly and given in [18]. For further analysis it is important to note that

$$\Omega^{+4} = -\frac{1}{16}(A^+)^2(\bar{A}^+)^2\mathrm{tr}\left(\frac{\cosh(s\mathcal{N})-1}{\mathcal{N}^2}\right)\mathrm{tr}\left(\frac{\cosh(s\bar{\mathcal{N}})-1}{\bar{\mathcal{N}}^2}\right). \tag{4.9}$$

One can show that only this last term in expansion of the exponent will survive in the coincidence limit $\theta^+=\theta^{+'}$ which should be taken in (4.1), since $(D^-)^4(D^+)^4\delta^8(\theta-\theta')|_{\theta=\theta'}=1$. All other terms with less then four (D^-) are killed at the coincident limit. As a result, we obtain, as the coefficient the maximally admissible number of the quantities $A^+,\bar{A}^+$ with non-zero Grassmann parity. Then all the other dependence on $A^+,\bar{A}^+$ in operator exponents (4.3) must be omitted and we get the expression for the effective action

$$\Gamma^{(1)}_v[V^{++},q^+] = \frac{i}{2}n(\Upsilon)\int d\zeta^{(-4)}e^{-s\mathcal{M}_v^2}K_{\mathrm{Sch}}(s)(A^+)^2(\bar{A}^+)^2\times \\ \times\mathrm{tr}\left(\frac{\cosh(s\mathcal{N})-1}{\mathcal{N}^2}\right)\mathrm{tr}\left(\frac{\cosh(s\bar{\mathcal{N}})-1}{\bar{\mathcal{N}}^2}\right), \tag{4.10}$$

where $K_{\mathrm{Sch}}(s)$ is the superfield Schwinger-type kernel [36, 13]. The latter is defined as follows $K_{\mathrm{Sch}}(x,x',s)=e^{-\frac{s}{2}\mathcal{D}^{\dot{\alpha}\alpha}\mathcal{D}_{\dot{\alpha}\alpha}}\{\mathbf{1}\delta^4(x-x')\}$. Now a computation of this heat kernel and its functional trace is standard (see e.g. [35, 15] for details). We write down only the final result

$$K_{\mathrm{Sch}}(s) = \frac{i}{(4\pi s)^2}\frac{s^2(\mathcal{N}^2-\bar{\mathcal{N}}^2)}{\cosh(s\mathcal{N})-\cosh(s\bar{\mathcal{N}})}.$$

Here $\mathcal{N}$ is given by $\mathcal{N}=\sqrt{-\frac{1}{2}D^4\mathcal{W}^2}$. It can be expressed in terms of the two invariants of the Abelian vector field $\mathcal{F}=\frac{1}{4}F^{mn}F_{mn}$ and $\mathcal{G}=\frac{1}{4}{}^\star F^{mn}F_{mn}$ as $\mathcal{N}=\sqrt{2(\mathcal{F}+i\mathcal{G})}$.

Relation (4.10) is a final result for the hypermultiplet dependent low-energy one-loop effective action of the Heisenberg-Euler type. We remind that the whole background hypermultiplet is concentrated in $\mathcal{M}_v^2$. The explicit form of it is:

$$\Gamma^{(1)}[V^{++},q^+] = \frac{1}{(4\pi)^2} n(\Upsilon) \int d\zeta^{(-4)} du \int_0^\infty \frac{ds}{s^3} e^{-s(\alpha^2(H)\mathcal{W}\bar{\mathcal{W}}+r(\Upsilon)q^{+a}q_a^-)} \times \tag{4.11}$$
$$\times \frac{\alpha^4(H)}{16}(D^+\mathcal{W})^2(\bar{D}^+\bar{\mathcal{W}})^2 \frac{s^2(\mathcal{N}^2-\bar{\mathcal{N}}^2)}{\cosh(s\mathcal{N})-\cosh(s\bar{\mathcal{N}})} \cdot \frac{\cosh(s\mathcal{N})-1}{\mathcal{N}^2} \cdot \frac{\cosh(s\bar{\mathcal{N}})-1}{\bar{\mathcal{N}}^2}\,.$$

It is easily to see that the integrand in (4.11) can be expanded in power series in the quantities $s^2\mathcal{N}^2$, $s^2\bar{\mathcal{N}}^2$. After change of proper time s to $s'\mathcal{W}\bar{\mathcal{W}}$ we get the expansion in power of $s'^2\frac{\mathcal{N}^2}{(\mathcal{W}\bar{\mathcal{W}})^2}$ and their conjugate. Since the integrand of (4.11) is already $\sim (D^+\mathcal{W})^2(\bar{D}^+\bar{\mathcal{W}})^2$, we can change in each term of expansion the quantities $\mathcal{N}^2$, $\bar{\mathcal{N}}^2$ by superconformal invariants Ψ^2 and $\bar{\Psi}^2$ [13] expressing these quantities from $\bar{\Psi}^2 = \frac{1}{\mathcal{W}^2}D^4 \ln \mathcal{W} = \frac{1}{2\mathcal{W}^2}\{\frac{\mathcal{N}^\beta_\alpha \mathcal{N}^\alpha_\beta}{\mathcal{W}^2} + \mathcal{O}(D^+\mathcal{W})\}$ and its conjugate. After that, one can show that each term of the expansion can be rewritten as an integral over the full $\mathcal{N}=2$ superspace.

It is interesting and instructive to evaluate the leading part of the effective action (4.11). Analysis of (4.11) (see the details in [18]) yields

$$\Gamma^{(1)}_{\text{lead}} = \frac{1}{(4\pi)^2} n(\Upsilon) \int d\zeta^{(-4)} du \frac{1}{16} \frac{D^+\mathcal{W}D^+\mathcal{W}}{\mathcal{W}^2} \frac{\bar{D}^+\bar{\mathcal{W}}\bar{D}^+\bar{\mathcal{W}}}{\bar{\mathcal{W}}^2} \frac{1}{(1-X)^2},$$

where

$$X = \frac{-q^{+a}q_a^-}{\mathcal{W}\bar{\mathcal{W}}} \frac{r(\Upsilon)}{\alpha^2(H)}\,. \tag{4.12}$$

As the next step, we rewrite this expression as:

$$\begin{aligned} &\frac{1}{(4\pi)^2}\int d\zeta^{(-4)} du \frac{1}{16}\Big\{ D^{+2}\ln\mathcal{W}\bar{D}^{+2}\ln\bar{\mathcal{W}} \\ &\quad + \sum_{k=1}^\infty \frac{1}{k^2(k+1)} D^{+2}\frac{1}{\mathcal{W}^k}\bar{D}^{+2}\frac{1}{\bar{\mathcal{W}}^k}\Big(-\frac{r(\Upsilon)q^{+a}q_a^-}{\alpha^2(H)}\Big)^k\Big\} \\ &\qquad = \frac{1}{(4\pi)^2}\int d^{12}z du \Big\{ \ln\mathcal{W}\ln\bar{\mathcal{W}} + \sum_{k=1}^\infty \frac{1}{k^2(k+1)} X^k \Big\}. \end{aligned} \tag{4.13}$$

That exactly coincides, up to group factor Υ with the earlier results [16–18]:

$$\Gamma^{(1)}_{\text{lead}} = \frac{1}{(4\pi)^2} n(\Upsilon) \int du\, d^{12}z \left(\ln\mathcal{W}\ln\bar{\mathcal{W}} + \mathrm{Li}_2(X) + \ln(1-X) - \frac{1}{X}\ln(1-X)\right). \tag{4.14}$$

Here $\mathrm{Li}_2(X)$ is the Euler's dilogarithm function. Next-to-leading corrections to (4.14) can also be calculated [18]. The remarkable feature of the low-energy effective action (4.14) is the appearance of the factor $r(\Upsilon)/\alpha(H)$ in argument X. This factor is conditioned by the vacuum structure of the model under consideration and depends on the specific features of the symmetry breaking. The form of the hypermultiplet dependent effective action analogous to (4.14) has been found originally in ref. [16] for $\mathcal{N}=4$ SYM theory and studied in refs. [17, 18] by different methods. In $\mathcal{N}=4$ SYM theory, the $\mathcal{N}=2$ vector multiplet and hypermultiplet both belong to the adjoint representation of the gauge group and the above factor in X is equal to 1. As a result, we conclude that the hypermultiplet dependent low-energy effective action has the universal form (4.14) for all $\mathcal{N}=2$ superconformal models, the difference of one $\mathcal{N}=2$ superconformal model from the others is conditioned only by factor $r(\Upsilon)/\alpha(H)$ in the quantity X (4.12). The same conclusion concerns also the general expression (4.10). Of course, the different models evidently contain the different common factors $n(\Upsilon)$ in front of integrals (4.10), (4.14).

Now we discuss some terms in the component Lagrangian corresponding to the effective action (4.14). Component structure of the effective action (4.14) has been studied [16] in the context of $\mathcal{N}=4$ SYM theory in bosonic sector for completely constant background fields $F_{mn}, \phi, \bar{\phi}, f^i, \bar{f}_i$. However, it was pointed out above that the superfield effective action (4.14) allows us to find the terms in the effective action up to fourth order in space-time derivatives of component fields. Now our aim is to find such terms in the hypermultiplet scalar component sector. To do that we omit all components of the background superfields besides the scalars $\phi, \bar{\phi}$ in the $\mathcal{N}=2$ vector multiplet and scalars $f, \bar{f}$ in the hypermultiplet and integrate over $d^4\theta^+ d^4\theta^- = (D^-)^4(D^+)^4$. Then we act these derivatives on the series under the integral in (4.13). To get the leading space-time derivatives of the hypermultiplet scalar components we should put exactly two spinor derivatives on each hypermultipelt superfiled. It yields, after some transformations, to the following term with four space-time derivatives on $q^\pm$ in component expansion of effective action (4.13):

$$\Gamma^{(1)}_{\rm lead} = \int d^4x du \frac{n(\Upsilon)}{(4\pi)^2} \sum_{k=2}^{\infty} \frac{1}{16} \frac{k-1}{k(k+1)} \frac{X^{k-2}}{(\mathcal{W}\bar{\mathcal{W}})^2} \times \tag{4.15}$$
$$\times \Big\{ -\bar{D}^{+\dot\alpha} D^{+\alpha} q^-_b \bar{D}^+_{\dot\alpha} D^-_\beta q^{+(b} \bar{D}^{-\dot\beta} D^{-\beta} q^{+a)} \bar{D}^-_{\dot\beta} D^+_\alpha q^-_a$$
$$+\frac{1}{2} \bar{D}^{+\dot\alpha} D^{+\alpha} q^-_b \bar{D}^{-\dot\beta} D^{-\beta} q^{+b} \bar{D}^-_{\dot\beta} D^-_\beta q^{+a} \bar{D}^+_{\dot\alpha} D^+_\alpha q^-_a$$
$$+\frac{1}{2} \bar{D}^{-\dot\beta} D^{+\alpha} q^-_b \bar{D}^{+\dot\alpha} D^{-\beta} q^{+b} \bar{D}^+_{\dot\alpha} D^-_\beta q^{+a} \bar{D}^-_{\dot\beta} D^+_\alpha q^-_a \Big\}|_{\theta=0} \,.$$

The straightforward calculation of the components[9] in this expression shows that among the many terms with four derivatives there is an interesting term of the special type

$$\Gamma^{(1)}_{\rm lead} = \frac{-1}{8\pi^2} n(\Upsilon) \left(\frac{r(\Upsilon)}{\alpha(H)} \right)^2 \left[\frac{X_0 - 2}{X_0^3} \ln(1 - X_0) - \frac{2}{X_0^2} \right] \times \tag{4.16}$$
$$\times \int du d^4x \frac{1}{(\phi\bar{\phi})^2} i\varepsilon^{\mu\nu\lambda\rho} \partial_\mu \tilde{q}^+ \partial_\nu q^+ \partial_\lambda \tilde{q}^- \partial_\rho q^- \,. \tag{4.17}$$

[9]Here we have used the relations $\bar{D}^+ D^+ q^- = \bar{D}^+ D^+ D^{--} q^+ = -\bar{D}^+ D^- q^+ = -2i\sigma^\mu \partial_\mu q^+$, as well as $\bar{D}^- D^- q^+ = 2i\sigma^\mu \partial_\mu q^-$, $\bar{D}^+ D^- q^+ = 2i\sigma^\mu \partial_\mu q^+$, $\bar{D}^- D^+ q^- = -2i\sigma^\mu \partial_\mu q^-$. In addition we have used $\int du u^+_i u^+_j u^-_k u^-_l = \frac{1}{6}(\epsilon_{ik}\epsilon_{jl} + \epsilon_{il}\epsilon_{jk})$.

As the first term in expansion over variable $X_0 = \frac{r(\Upsilon)\bar{f}^i f_i}{\alpha^2\bar{\phi}\phi}$ we have

$$\Gamma^{(1)}_{\rm lead} = -\frac{1}{48\pi^2}n(\Upsilon)\left(\frac{r(\Upsilon)}{\alpha(H)}\right)^2\int d^4x\frac{1}{(\phi\bar{\phi})^2}i\varepsilon^{\mu\nu\lambda\rho}(\partial_\mu\bar{f}^i\partial_\nu f_i\partial_\lambda\bar{f}^j\partial_\rho f_j - \partial_\mu\bar{f}^i\partial_\nu\bar{f}_i\partial_\lambda f^j\partial_\rho f_j)\,. \tag{4.18}$$

Here we have omitted all the terms containing the expressions of the type $\partial^\mu f\partial_\mu f$. The expression (4.16) has a form of the Chern-Simons-like action for the multicomponent complex scalar filed. The terms of such form in the effective action were discussed in refs. [32, 28] in context of $\mathcal{N}=4,2$ SYM models and in refs. [33] for $d=6, \mathcal{N}=(2,0)$ superconformal models respectively. Here the expression (4.16) is obtained as a result of straightforward calculation in the supersymmetric quantum field theory.

As the examples we list the values of $\alpha(H), r(\Upsilon)$ and $n(\Upsilon)$ for models considered in [9].

(**i**) $\mathcal{N}=4$ SYM theory with gauge groups SU(N), Sp($2N$) and SO(N). Here the hypermultiplet sector is composed of a single hypermultiplet in the adjoint representation of the gauge group. The background was chosen such that the gauge groups are broken down as follows SU(N) $\to$ SU($N-1$) $\times$ U(1), Sp($2N$) $\to$ Sp($2N-2$) $\times$ U(1), SO(N) $\to$ SO($N-2$) $\times$ U(1). All background fields aligned along element $H=$ U(1) of the Cartan subalgebra (with $\Upsilon = H$). The mass matrix becomes

$$(\mathcal{M}^2_v)_{IJ} = (\mathcal{W}\bar{\mathcal{W}} + \mathbf{q}^{+a}\mathbf{q}^-_a)(\alpha(H))^2\delta_{I,J}$$

and traces in eq. (3.11) produce the coefficient $n(\Upsilon)$ which is equal to the number of roots with $\alpha(H)\neq 0$, i.e. to the number of broken generators

$$n(\Upsilon) = \begin{cases} 2(N-1) & \text{for SU}(N)\,, \\ 4N-2 & \text{for Sp}(2N) \text{ and SO}(2N+1)\,, \\ 4N-1 & \text{for SO}(2N)\,. \end{cases}$$

The form of the mass matrix shows that in this case $r(\Upsilon) = \alpha(H)$.

(**ii**) The model introduced in [6]. The gauge group is USp($2N$) = Sp($2N,\mathbb{C}$) $\bigcap$ U($2N$). The model contains four hypermultiplets q^+_F in the fundamental and one hypermultiplet q^+_A in the antisymmetric traceless representation USp($2N$). The background fields $\mathcal{W}$, q^+_F, q^+_A are chosen to solve eqs. (2.6) with the unbroken maximal gauge subgroup USp($2N-2$) $\times$ U(1):

$$\mathcal{W} = \frac{\mathcal{W}}{\sqrt{2}}\,\mathrm{diag}(1,\underbrace{0,\ldots,0}_{N-1},-1,\underbrace{0,\ldots,0}_{N-1}), \quad q^+_F = 0\,,$$

$$(q^+_A)_\alpha{}^\beta = \frac{\mathbf{q}^+}{\sqrt{2N(N-1)}}\,\mathrm{diag}(N-1,\underbrace{-1,\ldots,-1}_{N-1},N-1,\underbrace{-1,\ldots,-1}_{N-1})\,.$$

The mass matrix $(\mathcal{M}^2_v)_{IJ}$ has been calculated in [9] and it has $n(\Upsilon) = 4(N-1)$ eigenvectors with the eigenvalue $\mathcal{M}^2_v = \bar{\mathcal{W}}\mathcal{W} + \frac{N}{N-1}\bar{\mathbf{q}}^j\mathbf{q}_j$.

(**iii**) The $\mathcal{N}=2$ superconformal model which is the simplest quiver gauge theory [7, 8]. Gauge group is SU$(N)_L\times$SU$(N)_R$. The model contains two hypermultiplets q^+, $\tilde{q}^+$ in the bifundamental representations $(\mathbf{N},\bar{\mathbf{N}})$ and $(\bar{\mathbf{N}},\mathbf{N})$ of the gauge group. In [9] a solutions

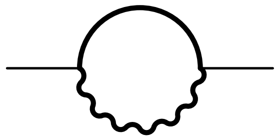

Figure 1: One-loop supergraph

of (2.6) with non-vanishing hypermultiplet components that specifies the flat directions in massless $\mathcal{N}=2$ SYM theories has been constructed. The moduli space of vacua for this model includes the following field configuration

$$\begin{aligned}\mathcal{W}_L=\mathcal{W}_R &= \frac{\mathcal{W}}{N\sqrt{2(N-1)}}\,\mathrm{diag}(N-1,\underbrace{-1\ldots,-1}_{N-1})\,,\\ q^+=\tilde{q}^+ &= \frac{\mathbf{q}^+}{\sqrt{2}}\,\mathrm{diag}(1,0,\ldots,0)\,,\end{aligned} \tag{4.19}$$

which preserves an unbroken gauge group $\mathrm{SU}(N-1)\times\mathrm{SU}(N-1)$ together with the diagonal $\mathrm{U}(1)$ subgroup in $\mathrm{SU}(N)_L\times\mathrm{SU}(N)_R$ associated with the chosen $\mathcal{W}$. In such a background the mass matrix has eigenvalue $\mathcal{M}_v^2=\frac{1}{N-1}\bar{\mathcal{W}}\mathcal{W}+\frac{1}{N}\mathbf{q}^{+a}\mathbf{q}_a^-$ and the corresponding $n(\Upsilon)=4(N-1)$.

5. Hypermultiplet dependent contribution to the effective action beyond the on-shell condition

In the above consideration, as well as in the papers on the hypermultiplet dependent effective action [16−18], a crucial point was the condition that the hypermultiplet q^+ satisfies the one-shell conditions (2.8) and the constraint $q^+=D^{++}q^-$. As it has been pointed out in [16] these conditions are sufficient to get all the leading low-energy contributions to the effective action. Here we relax the on-shell conditions and study some of possible subleading contributions with the minimal number of space-time derivatives in the component effective action.

We consider a supergraph given in figure 1 with two external hypermultiplet legs and with all propagators depending on the background $\mathcal{N}=2$ vector multiplet. Here the wavy line stands for the $\mathcal{N}=2$ gauge superfield propagator and the solid external and internal lines stand for the background hypermultiplet superfields and quantum hypermultiplet propagator respectively. For simplicity we suppose that the background field is Abelian and omit all group factors. The corresponding contribution to effective action looks like

$$\begin{aligned} i\Gamma_2 = &\int d\zeta_1^{(-4)}d\zeta_2^{(-4)}du_1du_2\left(\frac{(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4}{(u_1^+u_2^+)^3}\frac{1}{\widehat{\Box}_1}\delta^{12}(1|2)\right)\times\\ &\times\left(\frac{(\mathcal{D}_2^+)^4(\mathcal{D}_1^+)^4}{\widehat{\Box}_2\widehat{\Box}_1}\delta^{12}(2|1)(\mathcal{D}_1^{--})^2\delta^{(-2,2)}(u_2,u_1)\right)\tilde{q}^+(z_1,u_1)q^+(z_2,u_2).\end{aligned} \tag{5.1}$$

As usually, we extract the factor $(D^+)^4$ from the vector multiplet propagator for reconstructing the full $\mathcal{N}=2$ measure. Then we shrink a loop into a point by transferring the $\widehat{\Box}$ and $(\mathcal{D}^+)^4$ from first δ-function to another one and kill one integration. At this procedure the operator $\widehat{\Box}$ does not act on q^+ because we are interesting in the minimal number of space-time derivatives in the component form of the effective action. As a result, one obtains

$$i\Gamma_2 = \int \frac{d\zeta_1^{(-4)}du_1 du_2}{(u_1^+u_2^+)^3}\frac{(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4(\mathcal{D}_1^+)^4}{\widehat{\Box}_2\widehat{\Box}_1^2}\delta^{12}(z-z')\Big| \times \\ \times \Big((\mathcal{D}_1^{--})^2\delta^{(-2,2)}(u_2,u_1)\Big)\,\tilde{q}^+(z_1,u_1)q^+(z_1,u_2)\,. \qquad (5.2)$$

Further we use twice the relation (3.7) [14] allowing us to express the $(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4$ as a polynomial in powers of $(u_1^+u_2^+)$. Then after multiplying the $(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4(\mathcal{D}_1^+)^4$ with the distribution $1/(u_1^+u_2^+)^3$ we obtain a polynomial in $(u_1^+u_2^+)$ containing the powers of this quantity from 5-th to 1-st. The first order is just a contribution of the type which we considered in the previous section, because one derivation $(D^{--})^2$ is used for transformation $(u_1^+u_2^+)$ into $(u_1^+u_2^-)|_{u_1=u_2}=1$ in the coincident limit. Another D^{--} transforms q^+ into q^-. All that has been already done in section 4. Therefore, keeping only the first order in $(u_1^+u_2^+)$ we get a contribution including the combination q^+q^- without derivatives. As we pointed out in section 4, to obtain such a contribution it is sufficient to consider the hypermultiplet satisfying on-shell condition (2.8).

Here we consider the new contribution to the effective action containing term $(u_1^+u_2^+)^2$ in the above polynomial:

$$\frac{(\mathcal{D}_1^+)^4(\mathcal{D}_2^+)^4(\mathcal{D}_1^+)^4}{(u_1^+u_2^+)^3} = \qquad (5.3)$$
$$\ldots + (u_1^+u_2^+)^2(u_1^-u_2^+)(u_2^-u_1^+)(\mathcal{D}_1^+)^4\left(\frac{i}{2}\widehat{\Box}_1\Delta_2^{--}(u_2^+u_1^-) - \frac{i}{2}\Delta_1^{--}\widehat{\Box}_2(u_1^+u_2^-)\right) + \ldots$$

The ellipsis means the terms with the powers of $(u_1^+u_2^+)$ other then 2. One can show that in the coincident limit they disappear. Now transferring $(D^{--})^2$ on $(u_1^+u_2^+)^2$ we obtain the expression:

$$i\Gamma_2 = i\int d\zeta^{(-4)}du(\mathcal{D}^+)^4\frac{1}{\widehat{\Box}^3}(\underbrace{\widehat{\Box}\Delta^{--}}_{\Gamma_2(1)} - \underbrace{\Delta^{--}\widehat{\Box}}_{\Gamma_2(2)})\delta^{12}(z-z')|_{z=z'}\tilde{q}^+(z,u)q^+(z,u)\,, \qquad (5.4)$$

where Δ^{--} is defined in (3.8).

Let us consider each of the two underlined contributions separately. We use the representation

$$\frac{1}{\widehat{\Box}^2}\Delta^{--}\delta^{12}(z-z')| = \int ds\, s e^{s\widehat{\Box}}\Delta^{--}\delta^{12}(z-z')|, \qquad (5.5)$$

where $|$ means the coincident limit $z=z'$. Then we can apply a derivative expansion of the heat kernel. The goal is to collect the maximum possible number of factors of $\mathcal{D}^+,\mathcal{D}^-$ acting on $(\theta^+-\theta'^+)^4(\theta^- - \theta'^-)^4$ and having the minimum order in s in the integral over s.

Higher orders in s generate the higher spinor derivatives in the effective action. We take terms $\frac{1}{2}\mathcal{W}(\mathcal{D}^-)^2 + c.c.$ from Δ^{--} and expand the exponential so as to find $(\mathcal{D}^-)^4$. The eq. (5.5) allows us to write the leading contribution to $\Gamma_2(1)$ as follows

$$\Gamma_2(1) = -\int d^{12}zdu \int_0^\infty ds \cdot s \int \frac{d^4p}{(2\pi)^4} e^{-sp^2} e^{s(\mathcal{W}\bar{\mathcal{W}}-\varepsilon)} \frac{s^2}{32} \bar{\mathcal{W}}(D^{+\alpha}\mathcal{W}D^+_\alpha\mathcal{W}) \times \\ \times (D^-)^2(\bar{D}^-)^2\delta^8(\theta-\theta')|\tilde{q}^+q^+ + \text{ c.c.} \quad (5.6)$$

After trivial integration over p and s this contribution has the form

$$\Gamma_2(1) = \frac{i}{32\pi^2}\int d^{12}zdu \frac{1}{\bar{\mathcal{W}}}\frac{D^+\mathcal{W}D^+\mathcal{W}}{\mathcal{W}^2}\tilde{q}^+(z,u)q^+(z,u)(\mathcal{D}^-)^4\delta^8(\theta-\theta')| \\ + \frac{i}{32\pi^2}\int d^{12}zdu \frac{1}{\mathcal{W}}\frac{\bar{D}^+\bar{\mathcal{W}}\bar{D}^+\bar{\mathcal{W}}}{\bar{\mathcal{W}}^2}\tilde{q}^+(z,u)q^+(z,u)(\mathcal{D}^-)^4\delta^8(\theta-\theta')| \,. \quad (5.7)$$

Now we fulfil the same manipulations with the second underlined contribution $\Gamma_2(2)$ keeping the same order in s and $D^-, \bar{D}^-$ as in the expression (5.7):

$$\Gamma_2(2) = -\int d^{12}zdu\tilde{q}^+q^+ \int_0^\infty \frac{dss^2}{2}\int \frac{d^4p}{(2\pi)^4} e^{-sp^2+isp\mathcal{D}+s\widehat{\Box}} \times \quad (5.8)$$
$$\times(ip^{\alpha\dot{\alpha}}\mathcal{D}^-_\alpha\bar{\mathcal{D}}^-_{\dot{\alpha}} + \Delta^{--})(-\frac{1}{2}p^{\alpha\dot{\alpha}}p_{\alpha\dot{\alpha}} + ip^{\alpha\dot{\alpha}}\mathcal{D}_{\alpha\dot{\alpha}} + \widehat{\Box})\delta^8(\theta-\theta')|$$
$$= -\int d^{12}zdu\tilde{q}^+q^+ \int_0^\infty \frac{dss^2}{2}\int \frac{d^4p}{(2\pi)^4} e^{-sp^2+s\mathcal{W}\bar{\mathcal{W}}-\varepsilon s} \times$$
$$\times\frac{1}{2}\bar{\mathcal{W}}(\bar{D}^-)^2\frac{1}{4}(D^+\mathcal{W})(D^+\mathcal{W})\{-\frac{s^2}{4}p^{\alpha\dot{\alpha}}p_{\alpha\dot{\alpha}} + s\}\frac{1}{2}(D^-)^2\delta^8(\theta-\theta')| + (\mathcal{W}\leftrightarrow\bar{\mathcal{W}}) \,.$$

Integration over momenta in this expression[10] gives $\frac{i}{(4\pi s)^2}\{-\frac{s^2}{4}\frac{4}{s} + s\} = 0$. After that we see that the leading term of the form (5.7) is absent in $\Gamma_2(2)$. Then it is not difficult to show that the contribution (5.7) is rewritten as follows [we use $\int d^2\bar{\theta}^- = \bar{D}^{+2}$]

$$-\frac{i}{32\pi^2}\int d^4xd^4\theta^+d^2\theta^-du(\bar{D}^+)^2(D^+)^2(\ln\mathcal{W})\frac{1}{\bar{\mathcal{W}}}\tilde{q}^+(z,u)q^+(z,u)(\mathcal{D}^-)^4\delta^8(\theta-\theta')| + \text{ c.c.}$$

The non-zero result arises when all D^+ - factors act only on the spinor delta-function. Thus, the contribution under consideration is written as an integral over the measure $d^4xdud^4\theta^+d^2\theta^-$ which looks like "3/4 - part" of the full $\mathcal{N}=2$ harmonic superspace measure $d^4xdud^4\theta^+d^4\theta^-$.

Therefore, the hypermultiplet dependent effective action contains the term

$$\Gamma_2 = -\frac{i}{32\pi^2}\int d^4xdud^4\theta^+d^2\theta^-\frac{1}{\bar{\mathcal{W}}}\ln(\mathcal{W})\tilde{q}^+q^+|_{\bar{\theta}^-=0} \quad (5.9)$$
$$-\frac{i}{32\pi^2}\int d^4xdud^4\,\theta^+d^2\bar{\theta}^-\frac{1}{\mathcal{W}}\ln(\bar{\mathcal{W}})\tilde{q}^+q^+|_{\theta^-=0} \,.$$

Presence of such a term in the effective action for $\mathcal{N}=2$ supersymmetric models in subleading order was proposed in [28]. Here we have shown how this term can be derived in the supersymmetric quantum field theory.

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[10] $\int \frac{d^4p}{(2\pi)^4}e^{-sp^2} = \frac{i}{(4\pi s)^2}$, $\int \frac{d^4p}{(2\pi)^4}p_{\alpha\dot{\alpha}}p^{\beta\dot{\beta}}e^{-sp^2} = \frac{i}{(4\pi s)^2}\frac{1}{s}\delta^{\beta\dot{\beta}}_{\alpha\dot{\alpha}}$

It is interesting and instructive to find a component form of such a non-standard superfield action (5.9). Here we consider only a purely bosonic sector of (5.9). After integration over anticommuting variables, which can be equivalently replaced by supercovariant derivatives evaluated at $\theta = 0$, one gets:

$$\begin{aligned}\Gamma_2 = & \frac{i}{4\pi^2}\int d^4x du \frac{1}{\mathcal{W}\bar{\mathcal{W}}} D^+_\beta \bar{D}^-_{\dot\alpha}\tilde{q}^+ \bar{D}^{-\dot\alpha} D^-_\alpha q^+ D^{+\beta} D^{-\alpha}\mathcal{W}|_{\theta,\bar\theta=0} \\ & +\frac{i}{4\pi^2}\int d^4x du \frac{1}{\mathcal{W}\bar{\mathcal{W}}} \bar{D}^+_{\dot\beta} D^{-\alpha}\tilde{q}^+ D^-_\alpha \bar{D}^-_{\dot\alpha} q^+ \bar{D}^{+\dot\beta}\bar{D}^{-\dot\alpha}\bar{\mathcal{W}}|_{\theta,\bar\theta=0}.\end{aligned} \tag{5.10}$$

Since $D^-_\alpha \bar{D}^-_{\dot\alpha} q^+ = -2i\partial_{\alpha\dot\alpha} f^-$, $\bar{D}^+_{\dot\beta} D^-_\beta \tilde{q}^+ = 2i\partial_{\beta\dot\beta}\tilde{f}^+$ we have

$$\Gamma_2 = \frac{i}{\pi^2}\int d^4x du \frac{1}{\phi\bar\phi}\partial_{\beta\dot\alpha}\tilde{f}^+ \partial^{\dot\alpha}_\alpha f^- F^{\beta\alpha} + \frac{i}{\pi^2}\int d^4x du \frac{1}{\phi\bar\phi}\partial^\alpha_{\dot\beta}\tilde{f}^+ \partial_{\alpha\dot\alpha} f^- \bar{F}^{\dot\beta\dot\alpha}, \tag{5.11}$$

where $F^{\alpha\beta}, \bar{F}^{\dot\alpha\dot\beta}$ are the spinor components of Abelian strength F_{ab}. Then one converts the spinor indices into vector ones. As a result, we obtain a Chern-Simons-like contribution to the effective action containing three space-time derivatives

$$\Gamma_2 = -\frac{1}{2\pi^2}\int d^4x \frac{1}{\phi\bar\phi}\varepsilon^{mnab}\partial_m \bar{f}^i \partial_n f_i F_{ab} \ . \tag{5.12}$$

This expression is the simplest contribution to the hypermultiplet dependent effective action beyond the on-shell conditions (2.8) for the background hypermultiplet. Of course, there exist other, more complicated contributions including the hypermultiplet derivatives, they also can be calculated by the same method which led to (5.9). Here we only demonstrated a procedure which allows us to derive the contributions to the effective action in the form of integral over 3/4 - part of the full $\mathcal{N} = 2$ harmonic superspace.

6. Summary

We have studied the one-loop low-energy effective action in $\mathcal{N} = 2$ superconformal models. The models are formulated in harmonic superspace and their filed content correspond to the finiteness condition (1.1). Effective action depends on the background Abelian $\mathcal{N} = 2$ vector multiplet superfield and background hypermultiplet superfields satisfying the special restrictions (2.6), (2.7) which define the vacuum structure of the models. The effective action is calculated on the base of the $\mathcal{N} = 2$ background field method for the background hypermultiplet on-shell (2.8) and beyond the on-shell conditions.

We have shown that the hypermultiplet dependent one-loop effective action for the theory under consideration is associated with a special superfield operator (3.13), acting only in the sector of quantum vector multiplet superfields. The coefficients of this operator contain the background superfields and depend on details of gauge symmetry breaking. We prove that for evaluating the one-loop effective action it is sufficient to consider the simple case when the operator has the universal form (3.16).

The hypermultiplet dependent one-loop low-energy effective action is calculated in the form of an integral over the proper time. It was proved that to find the low-energy contributions to the effective action it is sufficient to consider on-shell vector multiplet and hypermultiplet. Final result for such a case is given by the relation (4.10) which is the $\mathcal{N}=2$ superfield analog of the Heisenberg-Euler effective action. The leading part of the low-energy effective action (4.14) has a general form[11] and depends on the quantity $X = \frac{-\mathbf{q}^{+a}\mathbf{q}^{-}_{a}}{\mathcal{W}\bar{\mathcal{W}}}\frac{r(\Upsilon)}{\alpha(H)}$ (4.12) containing the details of the vacuum structure of the model. Using the superfield effective action (4.14) we calculated the lowest space-time dependent terms in the sector of scalar hypermultiplet components. These terms contain four space-time derivatives of scalar fields and have a Chern-Simons-like form (4.16).

We studied possible contributions to the effective action which can be generated if we go beyond on-shell conditions (2.8) for the background hypermultiplet. The harmonic supergraph with two external hypermultiplet legs and with background vector multiplet dependent propagators has been computed and its leading low energy contribution has been found. We proved that the final result has a very interesting superfield structure and is written as an integral over 3/4 of the full $\mathcal{N}=2$ harmonic superspace (5.9). The presence of such terms in the effective action of $\mathcal{N}=2$ supersymmetric theories was recently proposed in [28]. We computed the component structure of the effective action (5.9) in the bosonic sector keeping the scalar components of the background hypermultiplet and vector component of the background $\mathcal{N}=2$ gauge multiplet. The result (5.12) has a Chern-Simons-like form and contains three space-time derivatives of the component fields.

To conclude, we have analyzed the general structure of the hypermultiplet dependent one-loop low-energy effective action of $\mathcal{N}=2$ superconformal models. For an on-shell hypermultiplet we found the universal expression for the effective active action. For hypermultiplet beyond on-shell, we calculated the special manifestly $\mathcal{N}=2$ supersymmetric subleading contribution which is written as an integral over 3/4 of the full $\mathcal{N}=2$ harmonic superspace. We believe that such contributions deserves a special study.

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